THE

Young *Trigonometer's* Compleat GUIDE.

Being the MYSTERY and RATIONALE OF

Plain TRIGONOMETRY Made clear and easy:

VOL. I. In TWO PARTS.

PART I. Containing a Proper Collection of *Definitions, Theorems* and *Problems*, requisite to the Art. Various Methods of constructing a *Canon* of *Natural Sines, Tangents* and *Secants*: and also of the *Logarithmetic Canon*. The Solution of all the *Cases* of a *Right-angled Plain Triangle*, in all its Varieties, by the following Methods;

I. By the *Logarithmetic Canon*.
II. By *Natural Sines, Tangents,* &c.
III. By the *Sliding Rule*.
IV. By *Scale* and *Compasses*.
V. By the *Sector*.
V[illegible] y *Geometrical Construction*.
VII. By [illegible] *Practical* [illegible]
VIII. Per [illegible]
IX. [illegible] *Arithmetick*.
[illegible]

With the Solution of Oblique [illegible] *Theorems* for finding the *Area* of [illegible]

PART II. Containeth the Applicatio[illegible] *[illegible]try*, to the following Mathematic[illegible]

I. *Navigation* in its several Kinds.
II. *Cosmography* and *Geography*.
III. *Astronomy*.
IV. *Fortification*.
V. The *Art of Gunnery*.
VI. [illegible]
VII. [illegible] *Longimetry*.
VIII [illegible]
IX. [illegible]
X. *Perspe*[illegible]

The Whole being conducted in a Method [illegible] different from any Extant; with great Variety [illegible] Art.

By *BENJAMIN M*[illegible]
AUTHOR of the *P*[illegible]*il*[illegible] *Grammar*.

LONDON: Printed for *J. Noon*, at the *White-Hart* in *Cheapside*, near *Mercers-Chapel*. 1736.

THE

PREFACE.

IN the Course of my Studies on this Branch of the Mathematics, *I observ'd much Obscurity, Irregularity and Deficiency in the Writings of most Authors on this Subject. I have therefore in the following Treatise endeavour'd to redress this Grievance, and to render this Science more plain, easy, and comprehensive; to which End I have laid down a large Number of* Definitions, Theorems, *and* Problems, *which are absolutely necessary to be learned. The* Definitions *inform us of the Nature, Names, and Differences of the several Lines and Figures used therein; and also other Terms of the Art. The* Theorems *contain all the Mystery of the Science, on a Demonstration and due understanding of which depends the Rationale of* Trigonometry; *that is, the Reason of all the various Operations of the Art; Thirty-one of the best* Theorems *I have collated for this Purpose, which serve more immediately for understanding the Reason of the Methods for making the* Canon *of* Natural Sines, Tangents, *and* Secants.

The Theorems *ought to be well studied. In demonstrating them I have taken a new Method, after the Manner of Mr.* Le Clerc, *in his* Practical Geometry, *which I think is the most clear, perspicuous and easy of all others. The* Problems *are adapted to the Capacity of all, enabling them to make a Geometrical Construc-*

tion of any Process or Operation in Trigonometry; *in which the Learner ought to be very perfect. In the next Place follow the several Methods of constructing a* Canon *of* Natural Sines, Tangents *and* Secants, *invented by the Learned, and here illustrated and exemplified in Numbers. This Matter however neglected in Books of* Trigonometry, *is of the last Importance to finish an Artist in this Part of Learning. Having learn'd this, the Reason of the* Logarithmetic Canon *is easy, as will appear in due Place. After having thus laid the Foundation Principles, I proceed to the Method of solving all the Six Cases of a* Right-angled Triangle, *in all the three Varieties, and that by ten several Methods; some by* Natural Numbers, *some by* Artificial Numbers, *and others by* Instruments.

I have shewn how the Method by Logarithms *depends on those by* Natural Numbers; *and how the Method by Instruments depends on both; I let the young Learner at once into the Mystery, or Theory and Practice of each Method, and have taken care to teach him nothing, for the which he may not be able to render a Reason. Therefore I have contriv'd a new Method to convince him of the Rationale of every Analogy, by* Logarithms; *or why we say* Sine, *or* Tangent, *or* Secant; *and why the Parts of Analogy are placed as they are, and the Reason of the large Indices to those Sort of Numbers; and have endeavour'd to make the whole clear and intelligible to the Learner by new Schemes for that Purpose.*

I have also describ'd the Nature and Use of the several Lines on the Instruments; as the Sector, Plain Scale, Sliding-Rule, *&c. as I have gone along; and have illustrated their Uses in the Doctrine of* Triangles. *You have also an Ample Collection of* Anomalous Cases *of a* Right Triangle, *with their Solution by* Algebraic Theorems.

Then

Then follows the Method of solving all the Cases of Oblique Triangles, *by the foresaid new Method, and Schemes; shewing the Rationale of every Process.*

And in the last Place you have a Variety of Theorems, *new and curious, for finding the Area of any* Plain Triangle, *by having given any of the* Sides, *or* Sides *and* Angles *together.*

In the second Part of this Work, I have applied the Doctrine of Plain Trigonometry *to the ten Mathematical Arts and Sciences mention'd in the Title-Page. In each of which I have given a General Account of the Art, its Principles and Maxims, the Nature and Properties of its several Parts, and the Use of* Trigonometry *in the whole: By which means the Reader will meet with a Kind of Epitome of the Mathematics, so far as it depends on* Plain Trigonometry: *Here are inserted a great Number of very rare and curious Propositions in* Cosmography, Geography, Astronomy, Gunnery, Mechanics, Surveying, Optics *and* Perspective, *which are not to be found in any one Author (that I know of) on this Subject, and yet such as properly appertain thereunto, the Reader may see the Particulars in the Table of Contents. One Thing I would acquaint the Reader with, that I have been concise, yet I think sufficiently prolix, in those Parts which are in every Book that treats of this Art,* viz. Navigation, Altimetry, Fortification, *and in some other Parts have perform'd the Operations by the Instrumental Methods, as by the* Gunter, *the* Sector, &c. *and therefore in such Cases, the Reader is not to expect such Mathematical Exactness as by* Logarithms, *because the Matter does not require it. Upon the whole, I have omitted nothing that I could think of which might render this Work a compleat Guide to the young* Trigonometer; *and I hope in the Use it will be found to be such.*

As

As for what belongs to Spherical Trigonometry *and its Application, that is contained in the second Volume.*

To conclude, having given a general Account of this Piece, its Occasion, *Subject and Method;* I *can not but expect the candid* Readers *will excuse small* Faults *that may have escaped my* Notice; *and wish them all the Profit in the perusal of it, which was intended in its Composure by him, who will never think much of any Pains whereby any* Benefit *may accrue to any of his* Fellow *Creatures.*

Benjamin Martin.

From my School in
Chichester, April 8.
1734.

The

The Contents of the Book.

PART I.

Chap.

PART II.

V. *To*

VII.

BOOKS Printed for J. Noon.

I. THE Philosophical Grammar; being a View of the Present State of experimented Physiology, or Natural Philosophy. In four Parts. Part I. *Somatology*, treateth of the universal Nature and Properties of Matter, or Substance, and the specifick Qualities of natural Bodies. Part II. *Cosmology*, exhibiting a general View of the Universe, and its great constituent Parts; the Sun, Moon, Planets, Comets, fix'd Stars, &c. Part III. *Aerology*, compriseth the Philosophy of the Atmosphere, shewing the wonderful Nature and Properties of the Air, Wind, Meteors, and other Phænomena therein. Part. IV. *Geology*, containeth a Philosophical View of the terraqueous Globe, in all its Parts and Productions; as Minerals, Metals, Stones, &c. The Laws of Fluids; the Sea, its Tides, &c. Of Rivers, Springs, &c. Of Vegetation, and the Nature of Plants, Trees, &c. Of the Parts of Animal Bodies; and a Survey of the Nature of Beasts, Birds, Fishes, Insects, Reptiles, Shell-Animals, &c. The whole extracted from the Writings of the greatest Naturalists of the last and present Age, treated in the familiar way of Dialogue, adapted purposely to the Capacities of the Youth of both Sexes; and adorned and illustrated with Variety of Copper-Plates, Maps, &c. Several of which are entirely New, and all easy to be understood. By *Benj. Martin.*

II. A New Compleat and Universal System or Body of Decimal Arithmetick: containing, I. The whole *Doctrine* of *Decimal Numbers*, not only the Plain and Terminate, but also such as *Repeat* or *Circulate ad Infinitum*; and a Plain but perfect *Management* of both, laid down and explained in all the *Fundamental Rules* of *Plain Arithmetick*, and by *Logarithms*. II. The Application and Use of *Decimal Arithmetick* in all the *Parts* or *Branches* of *Arithmetical Science*, viz. *Vulgar Arithmetick*, *Vulgar Fractions*, *Duodecimal* and *Sexagesimal Arithmetick*; also in *Algebra* and *Logarithms*. In all which its *Excellency* and absolute *Necessity* is fully evinced. III. Its Application and Use in all such *Parts* of the *Mathematicks* as absolutely require its *Assistance*, viz. *Plain Trigonometry*, and the Arts depend-

ing

ing thereon; as, *Navigation, Fortification, Altimetry, and Longimetry:* Alſo the *Menſuration* of all Kinds of *Superficies and Solid Bodies*; and the Arts reſulting therefrom; as, *Gauging, Surveying*, &c. IV. A *New and Compleat Sett of Decimal Tables*, never before publiſhed; ſhewing by Inſpection the Value of all Kinds of Decimals (without the tedious Methods of Reductions hitherto uſed) to four or ſix Places of Figures: Alſo all the *Common Tables*, very much *inlarged, corrected, and improved*; wherein all the *Circulating Numbers are marked.* With all other Tables of Intereſt, Annuities, Exchange, *&c. neceſſary* to render the Work *compleat.* V. An exact and accurate *Canon of Logarithms* for natural Numbers. And thro' the whole, ſeveral Things new and uſeful, not here expreſs'd. By *Benj. Martin.*

III. The Young Student's Memorial Book, or Pocket Library: containing, I. The Rudiments of Logarithms, Decimals, and Algebra, in great Variety of Rules and Caſes. II. A very large Collection of Theorems and Canons for ſolving *Queſtions* and *Problems* in the various Parts of Arithmetic, Algebra, and Fluxions. III. Rules, Theorems and Canons, reſolving all the uſeful and common Problems in the Mathematical and Mechanical Arts and Sciences, viz *Menſuration, Gauging, Conics, Plain and Spherical Trigonometry, Navigation, Fortification, Gunnery, Aſtronomy, Dialling, Optics, Perſpective, Architecture, Mechanics, Geometry*, &c. IV. A large Collection of Mathematical Tables; ſerving to various Purpoſes of *Arithmetic, Aſtronomy, Geography, Chronology*, &c. with a perpetual Almanack, adjuſted to the preſent Year, 1735. V. An *Apparatus* of the common Mathematical Lines, for the Operation of any *Queſtion* or *Problem*, as is perform'd by the Plain Scale, *&c.* By *Benj. Martin.*

IV. A Paraphraſe, with Notes, on the Revelation of St. *JOHN.* Which compleats the Paraphraſe on the *New Teſtament* in the Manner of Dr. *Clarke.* By *Thomas Pyle*, A.M. Miniſter of *Lynn Regis* in *Norfolk.*

Bleſſed is he that readeth, and they that hear the words of this propheſy, and keep thoſe things which are written therein——! *Revel.* i. 3.

An

An Explication of certain Characters.

AS the Uſe of too many Characters or Abbreviations, does very much obſcure the Work, and perplex young Beginners; and without any at all, it would be prolix and verboſe, and both the Book and the Price muſt be thereby neceſſarily (tho' unreaſonably) increaſed; I have therefore made uſe of a few Characters, and thoſe ſo plain and eaſy, and withal ſo common in all Mathematical Books, that not for Brevity's ſake only, but for the ſake of reading and underſtanding any other Mathematical Treatiſe; the young Learner is obliged to be well acquainted with their Forms and Significations, which are as follows.

Characters.		*Significations.*
$+$	Signifieth	More; in Addition.
$-$		Leſs; in Subſtraction.
$\times$		Multiplied into; in Multiplication.
$\div$		Divided by; in Diviſion.
$=$		Equal to.
$\surd$		The Root of; as $^2\surd$, is the Square Root, &c.
q		Squared; as Aq, is A ſquared.
:		Is to; in Proportion.
: :		So is; in Proportion.
s		Sine; as sB, is the Sine of B.
t		Tangent;
se		Secant.

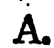
If there be any more, which I do not think of, the Reader will find them explained in their Places, in the Body of the Work.

CHAP.

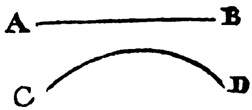
CHAP. I.

Of Geometrical Definitions.

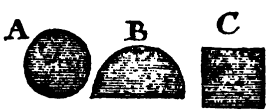
I. A *Point*, Geometrically consider'd, is the least assignable Part of Space, and absolutely indivisible; and is denoted by a Tittle; as at A.



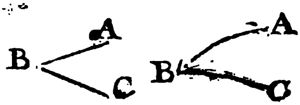
II. A *Line*, either strait or crooked, is a Length without Breadth or Thickness; and is generated by the Motion of a Point; as AB, CD.

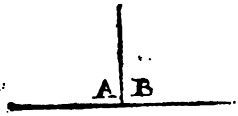


III. A *Superficies* or *Surface*, is Length and Breadth without Thickness, and terminated by one or more Lines; as A, B, C.

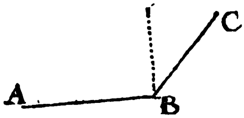


IV. An *Angle* is the Inclination or Meeting of two Lines in one Point; or the Space included between them; as the Angle A, B, C.



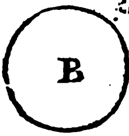


V. A *Right-Angle* is included between two Lines perpendicular to each other; as the Right Angle A, cr B.

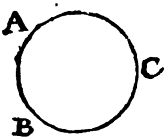


VI. An *Obtuse Angle* is greater than a Right Angle; as the Angle A B C.

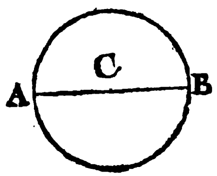
VII. An *Acute Angle* is less than a Right Angle; as the Angle D E F.



VIII. A *Circle* is a plain Superficies, whose Area is limited by perfect Round Line; as the Figure B.



IX. The *Periphery* or *Circumference* of a Circle is the Round Line that bounds it; A, B, C.

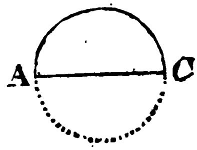


X. The *Center* of a Circle is the Point C, every way equally distant from the Periphery.

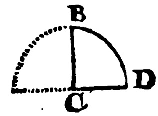
XI. The *Diameter* of a Circle is a Line passing through the Center, and ending at the Periphery on each Side; as the Line A B.

XII.

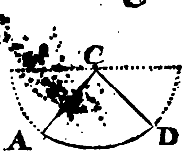
XII. A *Semicircle* (*i. e* half a Circle) is a Figure contained between the Diameter and half the Periphery of a Circle; as A B C.



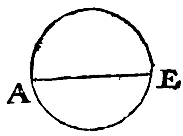
XIII. A *Quadrant* is one Quarter of a Circle, or half the Semicircle; as the Figure BCD.



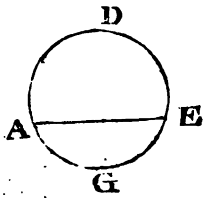
XIV. The *Sector* of a Circle is a Figure included between two Semidiameters and an Arch of the Periphery; as ACD.



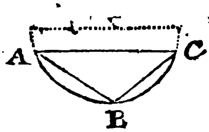
XV. A *Chord Line*, or *Subtense* of an Arch is any Right Line dividing the Circle into two unequal Parts, and is less than the Diamiter; as A E.

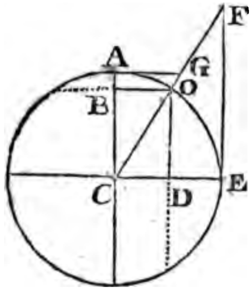


XVI. A *Segment* of a Circle is the Figure included between the Chord Line and Arch of the Circle, and is greater or less than a Semicircle; as the Figure ADE, or AGE.



XVII. An *Angle* in the Segment of a Circle is that which toucheth the Periphery, and is subtended by a Chord Line; as either of the three Angles A, B, or C.





XVIII. The *Radius* of a Circle, or Semidiameter, is half the Diamiter; as A C, or C E.

XIX. The *Measure* of an Angle, as the Angle OCE, is the Arch of the Circle OE.

XX. The *Sine* of that Angle, is the Line O D.

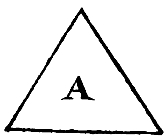
XXI. The *Tangent* thereof, is the Line E F.

XXII. The *Secant* thereof, is the Line C F.

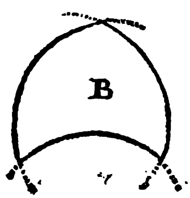
XXIII. The *Versed Sine* thereof, is the Line D E.

XXIV. The *Co-Sine*, *Co-Tangent*, and *Co-Secant*, are the Sine, Tangent, and Secant of the Complement of that Arch to a Quadrant, *viz.* of the Arch A O; as BO, A G, and C G.

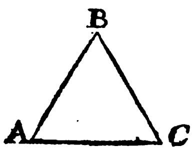
XXV. A *Triangle* is a Figure bounded by, or included between three Lines, and having three Angles.



XXVI. A *Plain* Triangle is terminated by three strait or Right Lines; as the Figure A.



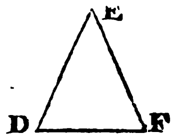
XXVII. A *Spherical* Triangle is that whose Sides are Parts or Arches of three Great Circles of the Sphere or Globe; and are each less than a Semicircle; as the Figure B.



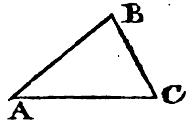
XXVIII. An *Equilateral* Triangle hath all its three Sides; as the Figure ABC, for AB=BC =AC.

XXIX.

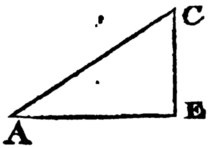
XXIX. An *Isoceles* Triangle hath only two Sides equal; as the Figure DEF. That is, DE =EF, (DF being greater or leſs.)



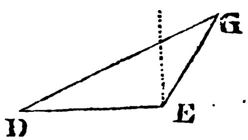
XXX. A *Scalenous* Triangle is that which hath all its three Sides unequal; as ABC.



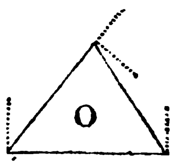
XXXI. A *Right-angled* Triangle is that which hath one Right Angle; as AEC, Right-angled at E.



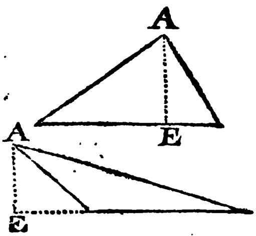
XXXII. An *Obtuſe-angled* Triangle is that which hath one Angle Obtuſe, (this is alſo call'd an *Amblygon.*) As DEG.



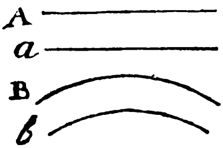
XXXIII. An *Acute-angled* Triangle is that which hath its Angles acute, (this is alſo call'd an *Oxygon.*) As the Figure O.



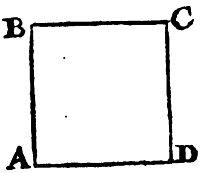
XXXIV. The *Height* or *Altitude* of any Plain Triangle is the Length of a Perpendicular let fall from any one Angle the oppoſite Side; and this Perpendicular may fall within or without the Triangle; as AE.



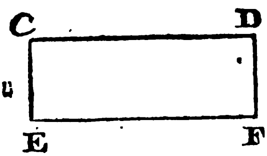
XXXV.



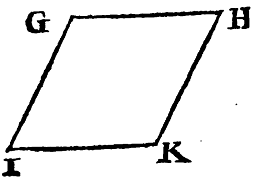
XXXV. *Parallel Lines*, whether strait or curved, are such as are equally distant in all their Parts, though infinitely extended; as A*a*, or B*b*.



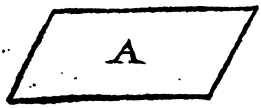
XXXVI. A *Square* is a plain regular Figure contained within four equal and parallel Lines or Sides, perpendicular to each other; and thereby making four Right Angles; as the Figure ABCD.



XXXVII. A *Rectangle* or *Parallelogram* (call'd also an *Oblong* or *Long Square*,) is a Figure that hath four Right Angles, and its two opposite Sides equal and parallel; as CDEF.



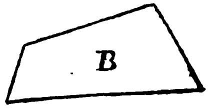
XXXVIII. A *Rhombus* is a Figure that hath four equal Sides; but two opposite Angles acute, the other two obtuse; as the Figure GHIK.



XXXIX. A *Rhomboides*, or Oblique-angled Parallelogram, a Figure whose two opposite Sides are equal and parallel, but its Angles the same as in the *Rhombus*, as the Figure annexed A.

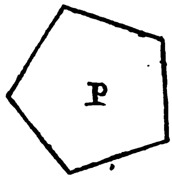
XL.

XL. A *Trapezium* is a Figure of four Sides; but all its Sides and Angles are unequal; as the Figure B.

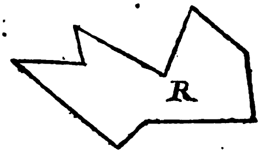


XLI. A *Polygon* is any Figure that hath more than four Sides; and is call'd,

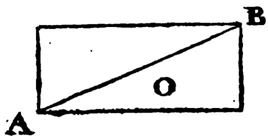
XLII. A *Regular Polygon* when all its Sides and Angles are equal; of which there are several Sorts, as the Pentagon P, &c.



XLIII. But an *Irregular Polygon* hath all its Sides and Angles unequal; as the irregular Figure R.



XLIV. A *Diagonal* is a Right Line drawn from the opposite Corners a Figure; A B, in the Figure O.



CHAP. II.

xioms; Postulata *or* Demands; *some Geometrical Terms explain'd.*

Axiom I.

qual Quantities be added to Equal Quantities, Sum of those Quantities will be Equal.

Thus,

Thus, if to the Equal Quantities	8= 8;
You add the Equal Quantities ——	6= 6;
Their Sums will be Equal———	14=14.

Axiom II.

If Equal Quantities be deducted from Equal Quantities, the Remaining Quantities are Equal.

Axiom III.

If Equal Quantities be multiplied by Equal Quantities, their Products will be Equal.

Axiom IV.

If Equal Quantities be divided by Equal Quantities, their Quotients will be Equal.

Axiom V.

Those Things which are Equal to one and the same Thing, are Equal to one another.

Axiom VI.

Those Things which are Double, Triple, *&c.* of the same or Equal Things, are Equal among themselves.

Axiom VII.

The Whole of any Thing is Greater than any Part thereof.

Axiom

Axiom VIII.

Every whole Thing is Equal to all its Parts taken together.

Axiom IX.

Those Things which being laid one upon another, do agree, or meet in all their Parts, are Equal to one another.

Note, The Converse hereof is true in Lines and Angles universally; but in Similar Figures only.

Postulata, or Demands.

I. It must be granted, That a Right Line may be drawn from one Given Point to another.

II. That a Right Line may be Produced, or made Longer from both Ends.

III. That upon any Given Point, or Center, and with any Given Distance, or Radius, a Circle may be described.

Geometrical Terms.

A *Proposition* is whatsoever is offered or proposed to be Done, or Demonstrated, in Geometry.

A *Problem* is that which proposeth something to be Done or Perform'd by the Rules of Art.

A *Theorem* is that which proposeth something to be Demonstrated, or made appear to be really true.

A *Demonstration* is an absolute, infallible and evident Proof of a Proposition, which proceeds from a Chain of Reasoning or Argumentation, drawn from such Plain Axioms and Self-evident Truths as cannot be denied with Reason.

A *Corollary* is some notable Truth following from, or gained by any foregoing Demonstration.

A *Consectary* is the same as a Corollary.

A *Lemma* is a Demonstration of something promised, in order to expedite and facilitate the Demonstration of the Theorem depending.

A *Scholium* is a learned or useful Remark or Commentary on some preceedent Discourse.

As the foregoing Definitions and Axioms will be frequently mention'd and refer'd to in the two next Chapters of Theorems and Problems, 'twill be very necessary, that the Learner should have them truly and firmly imprinted in his Memory before he goes farther; for unless he perceives the Reason of Things, the Practice will be but Confusion and Mistery to him. And as the Theory naturally preceeds the Practice; I shall next demonstrate a Collection of the useful and necessary Theorems, after a new and easy Manner.

Chapter III.

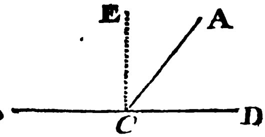
Geometrical Theorems.

Theorem I.

ONE strait Line AC standing on another strait Line BD, makes either two Right Angles, or two Equal to two Right ones.

Demon-

Demonstration.



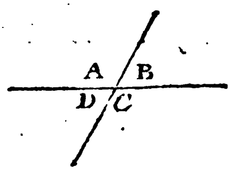
If the Line ——— —— AC
Be Perpendicular to —— BD
Then the Angle BCA=ACD
And each a Right one by *def.* 5.
If not; draw the Perpend. EC
Then will the two Angles ECA+ACD
Be equal to one Right Angle by *def.* 5.
But the Angle ———————— BCE
is a Right one; Therefore BCA+ACD
are equal to two Right Angles. *Q. E. D.*

Corollary. Hence if several strait Lines meet or stand together at one Point, and on the same Side of a Right Line, all the Angles taken together will be equal to two Right Angles.

Theorem II.

If a Right Line cut or cross, another Right Line, 'twill make the opposite Angles equal one to another.

Demonstration.

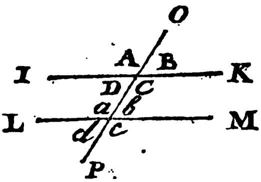


The Angles ———— A+B,
Are equal to two Right Angles —— ——— *Theo.* I.
So also are the Angles — B+C;
Then are the Angles A+B=B+C ——— ——— ——
Substract, on both Sides the Angle B,
There will remain (by Axiom 2.) A=C
After the same manner 'tis proved B=D
Therefore the opposite Angles are equal. *Q. E.D.*

Theorem III.

If a Right Line cross two Parallel Lines, 'twill make the alternate and opposite Angles equal.

Dimonstration.

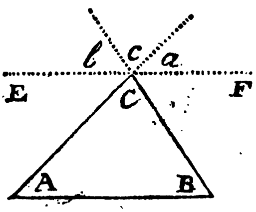


The Line ———— I K
Being parallel to the Line L M, —— —— —— }
The Inclination of —— O P
Must be equal to both I K and L M ——— }
Consequently the Angle B=b ———— }
But (by *Theor.* II) the Angle —— —— B=D
Then (by Axiom 5.) the Alternate Angles are equal ———— *viz.* } D=b
Therefore also —— ——— ——— —— C=a
Consequently all the opposite and alternate Angles are equal —— } *Q. E. D.*

Theorem IV.

The Three Angles of every Plain Triangle, are together equal to two Right Angles,

Demonstration.



Since the Line ——— E F
Is parallel to the Base —— A B
Then (by Theor. III) the Angle ———— A=a }
And the Angle —— B=b
Also (by Theor. II.) the Angle ———— C= c }
That is, the Angles —— A+B+C=a+b+c

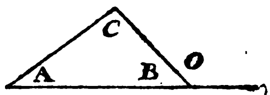
But

But (by Theor. I.) the Angles —— —— $a+b+c$
Are = two Right ones. Therefore the three Angles of the Triangle —— —— —— . $A+B+C$
Are (by Axiom 5) equal to two Right Angles *Q.E.D*

Theorem V.

If one Side of any plain Triangle be produced beyond the Triangle, The outward Angle will always be equal to the two inward and opposite Angles.

Demonstration.



By Theor. IV. The Three Angles of the Triangle $A+B+C$ —— —— Are equal to two Right Angles.
Alſo (by Theor. I.) the Angles $B+O$ are equal to the two Right Angles.
Subſtract the common Angle —— —— B
There will (by Ax. 2) remain $A+C=O$
That is, the two inward and oppoſite Angles = to the outward Angle. —— *Q. E .D.*

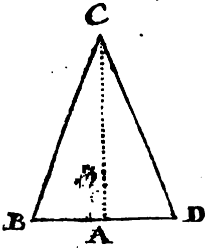
Cor. Hence, if one Angle of any plain Triangle be given, the Sum of the other two are alſo given.

Theorem. VI.

In every plain Triangle, Equal Sides ſubtend (or are oppoſite to) Equal Angles; and the contrary.

Demon-

Demonstration.



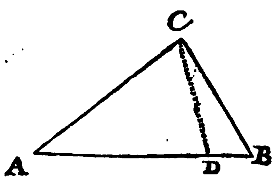
Suppose the Side or Leg — B C
Equal to the Side or Leg - C D,
Drop the Perpendicular — C A
Then are the Angles A B C + B C A — — — } Equal to one Right Angle by Theor. IV. — — — }
So also are the Angles ACD + A D C — — — — }
That is (by Axoim 5) — $\frac{1}{2}$ C+B = $\frac{1}{2}$ C+D
Substract the Common Angle from both Sides the Equation. — — — } *viz.* $\frac{1}{2}$ C
There remains (by A[illegible] the Angle — B=D
Therefore Equal Sides subtend equal Angles; *Q.E.D.*

Corol. Hence if the three Sides of any Triangle be equal, the three Angles will be equal to one another.

Theorem VII.

In every plain Triangle, the greatest or longest Side subtends the Greatest Angle; and *vice versa*, the Greatest Angle is subtended by the longest Side.

Demonstration.



From the Line — — A B
Strike off, or make AD=AC
Join the two Points C and D
Then (by Theor. VI.) is ACD=ADC, }
But (by Theor. 5) the Angle ADC=ABC+DCB }

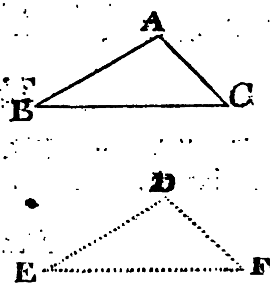
That

That is, the Angle ——— ——— ——— ACD
is Greater than the Angle ——— ——— ABC,
And therefore (by Axiom 7.) the Angle ACB
is Greater than the Angle ——— ——— ABC.
Thus tis proved the Angle ——— ——— ACB
is Greater than the Angle ——— ——— A,
Therefore the Greatest Side ——— ——— AB
Subtends the Greatest Angle —— ACB. *Q. E. D.*

Theorem VIII.

If one Triangle ABC, hath two Sides AB and AC, and an Angle included A, severally equal to two Sides DE, DF, and an Angle included D, of any other Triangle EDF; then are the rest of the Parts, and consequently both the Triangles equal.

Demonstration.



Lay the Point ——— D
On the Point ——— A
And the Line ——— ED
On the Line or Side — AB
Then shall the Points E & F
fall on the Points — B & C
And the Line or Side — DF
will fall on the Side — AC
Therefore (by Axiom 9) the Angle —— E=B
And also the Angle F=C. and DF=AC,
And consequently the whole Triangle — EDF
Is equal to the whole Triangle — BAC. *Q. E. D.*

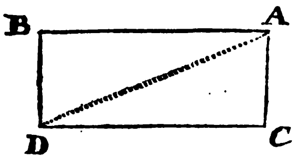
Corol. The same Thing happens if a Triangle has two Angles and a Side included, or all three Sides, equal to the same in any other Triangle.

Theorem

Theorem IX.

The oppoſite Sides AB, CD, of a Parallelogram are Equal, and the Diameter A B divides it into two equal Parts.

Demonſtration.

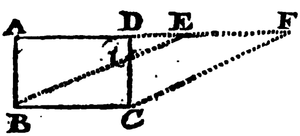


Becauſe the Side (Def. 37) AB ——— — — Is parallel to the Side CD
Therefore (Theo. III) the Angle —— ADC Is equal to the Angle BAD
So alſo the Angle —— —— —— DAC=ADB
And the Side included is —— —— —— AD,
Therefore (*Corol.* Theor. VIII) the Triangle - ADB,
Is equal to the Triangle ——— DAC ABC;
And becauſe of that, the Sides ——— AB = DC
And the Sides ——— AC = BD. *Q. E. D.*

Theorem X.

Parallelograms BD, BF on the ſame Baſe BC and betwixt the ſame Parallels, are equal to each other.

Demonſtration.



By Theor IX. the Sides AB=DC. ——— Alſo the Sides AD=BC =EF ——— ———
And the Side —— DE is common; therefore (*Ax.* I.) ——— AE=DF.
Alſo the Angle ——— ——— EAB=FDC:

Therefore

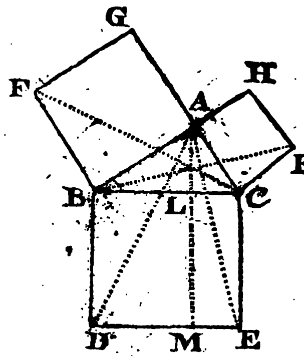
Therefore the Triangle ——— EAB=FDC
Subduct the common Triangle —— —— DGE
there remains (*Ax.* II) the Trapezium CGEF
equal to the Trapezium —— ——— ADGB
add the common Triangle —— —— BGC
Then will the Parallelogram —— —— ABCD
be equal to the Parallelogram —— —— EBCE.

Corol. The ſame is true of Triangles, ſince they are Halves of equal Parallelograms, by *Axiom* VI.

Theorem XI.

In any Right Angled Triangle AB, the Square of the Hypothenuſe BC, is equal to the Squares of the other two Sides AB and AC.

Demonſtration.



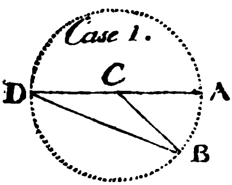
Becauſe the Angles DBC=FBA———
add the common Angle ABC ———
then will the Angle FBC=ABD———
But (by Def. 36.) the Sides AB=FB, and BD=BC.
Therefore (by *Theor.* VIII) the Triangle ABD is equal to the Triangle FBC ———
And (by *Cor.* to *Theor.* X.) the Parallelogram BM =2ABD —— —— —— —— ——
and the Parellelogram ——. —— BG=2FBC
Therefore (by *Ax.* 6.) the Parallelograms BM=BG
Thus tis proved the Parallelograms — CM=CH
Therefore (by *Axiom* I.) the whole Square BE = BG+CH. *Q. E. D.* —— —— ——

Corol. By this moſt noble and uſeful Theorem, firſt Invented by *Pythagoras*, is deduced the Method of Adding and Subſtracting Squares, Parallelograms, Circles, *&c.*

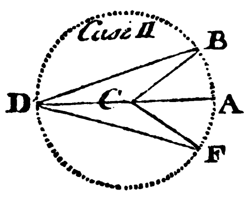
Theorem XII.

An Angle at the Center of any Circle is always double to the Angle at the Circumference, when both the Angles ſtand on the ſame Arch.

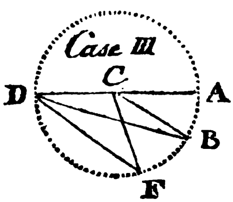
Demonſtration.



Caſe I. By *Theor.* V. The Angle ACB = D + B —— ——
and the Sides —— DC = BC
Therefore (by *Theor.* VI.) the Angle B = D. —— ——
Conſequently the Angle ACB = 2 D. *Q. E. D.* —— ——



Caſe II. By Caſe I. The Angle BCA = 2BDA —— ——
And the Angle FCA = 2FDA
But the Angles BCA + FCA = BCF —— —— ——
And 2 BDA + 2FDA = 2BDF.
Conſequently the Angle BCF = 2BDF. *Q. E. D.* ——



Caſe III By Caſe I. The Angle FCA = 2FDA —— ——
And the Angle BCA = 2BDA
But the Angles FCA — BCA = FCB —— —— ——
And 2FDA — 2BDA = 2FDB
Conſequently the Angle FCB = 2FDB. *Q. E. D.* ——

Corl.

Corol. I. Hence all Angles at the Periphery ſtanding on the ſame, or equal, Arches of a Circle are equal.

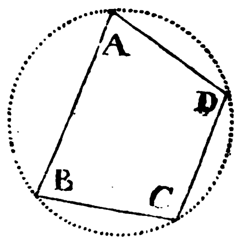
Corol. II. An Angle at the Periphery contains but half the Number of Degrees of the Arch it ſtands on.

Corol. III. And therefore an Angle in a Semicircle muſt be a Right one.

Theorem XIII.

The oppoſite Angles B and D, or A and C, of every four Sided Figure inſcribed in a Circle are equal to two Right Angles.

Demonſtration.

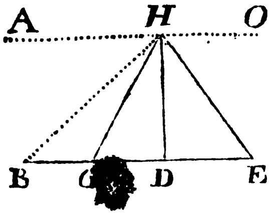


By *Corol.* II. of *Theor.* XII.
The Angle B —— ——
is half the Arch it ſtands on
ADC ——— ——— ———
Alſo the Angle —— —— D
is half the Arch it ſtands on
ABC —— —— —— ——
But both the Arches, *viz.*
ABC+ADC —— ——
make the whole Circle; *Ergo* ——— B+D
are half the Circle, or two Right Angles. *Q. E. D.*

Theorem XIV.

Triangles between the ſame or Equi-diſtant Parallels are to one another as their Baſes.

Demonstration.



Suppose the Bases BC = CD = DE ——— ———
And the two Parallels AO and BE ——— ———
Then (by *Theor.* X.) the Triangles BHC, CHD, DHE ——— ———
are all equal; hence the Triang. ——— CHE
is double to the Triangle ——— ——— DHE
Also the Base ——— ——— ——— ——— CE = 2DE
So also is the Triangle ——— ——— BHE
treble to the Triangle ——— ——— DHE
And the Base ——— ——— ——— BE = 3DE
Therefore, As the Triangle ——— BHE
is to the Triangle ——— ——— ——— DHE
So is the Base ——— ——— ——— ——— BE
to the Base ——— ——— ——— ——— DE. *Q. E. D.*

Corol. The same Thing is true of Parallelograms, they being Double of their Triangles.

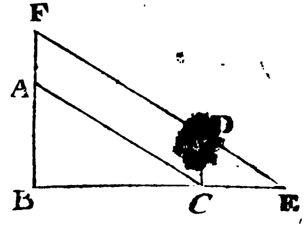
Theorem XV.

If two Triangles are Similar or Like, their like (or corresponding) Sides will be Proportional to each other.

Note; Similar Triangles are those which have the same single Angles severally equal to each other.

Demon-

Demonstration.



Let the Triangle ABC be similar to the Triangle DCE. ——— ——— ———

Because (by *Hypoth*) the Angle B = DCE ———

The Sides (by *Theor.* III) BF and CD ——— are Parallel. Also because BCA = DCE ———

Those Sides likewise ——— ——— ——— AC, and EF are Parallel. Therefore the Figure ——— ACDF is a Parallelogram. Whence (*Theor.* IX.) AF=CD, so also are the Sides ——— ——— ——— AC = FD

Therefore it will be, AB : AF (=CD) : : BC : CE.
And by Permutation; ——— AB : BC : : CD : CE.
Thus tis proved ——— ——— ——— BC : AC : : CE : DE.
And by Equality AB : AC : : CD : DE. *Q. E. D.*

Note. This latter Part of the Demonstration which I have braced, depends on the 2d Prop of *Euclid's* 6th Book, and the 16th, and 22d of the 5th Book; which tis proper to consult for Satisfaction; tho' they could not be conveniently here inserted, and yet I thought best to give *Euclid's* own Demonstration, as it is most Mathematical.

Scholium.

On this Noble *Theorem* depends the whole Doctrine of both Plain and Spherical-Trigonometry; and consequently Astronomy, Dialling, Navigation, Fortification, Surveying, Optics, and other Mathematical Arts, are but as it were a due Application hereof, as Mr. *Ward* has observ'd.

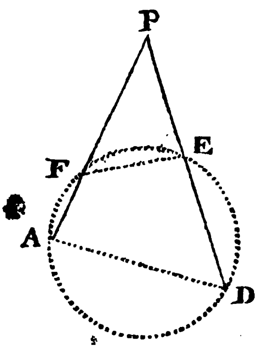
Hence

Hence alſo is deduced the Method of finding proportional Lines.

Theorem XVI.

If from a Point without a Circle, as P, be drawn 2 right Lines PA, PD, to the oppoſite Part of the Periphery, the Rectangle of one whole Line and its Part without the Circle, is equal to the Rectangle of the other whole Line and its Part without the Circle.

Demonſtration.



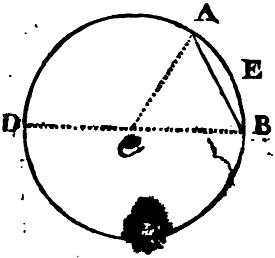
By *Theor.* I. and XIII. The Angle PFE=D —— they being both Complements of the Angle AFE to a Semicircle, and the Angle P —— —— —— is common; therefore the Triangle PFE —— —— is Similar (*Theor.* XV.) to the Triangle PDA. —— Therefore as AP : EP :: DP : FP —— —— ——

Conſequently —— $AP \times FP = DP \times EP$, *Q. E. D.*

Theorem XVII.

The Chord of 60 Degrees of the Arch of any Circle is equal to the Radius, or Semidiameter of that Circle.

Demon-



Demonstration.

(*Note.* Every Circle is supposed to be divided into 360 equal Parts, call'd Degrees.)

Suppose the Arch of 60 Degrees be AEB;
And the Chord thereof be AB;
Now because the Sides AC=CB,
Therefore (by *Theor.* VI.) the Angle A=B;
But (by *Theor.* V.) they are both equal to ACD, *viz.* 180 deg.—60 deg.=120 Degrees.
Therefore all the Angles are equal, ACB=B=A.
Consequently (by *Theor.* VI. *Cor.*) all the Sides are equal, *viz.* AC=AB=BC. *Q.E.D.*

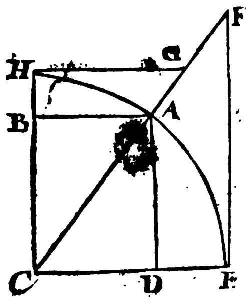
Corol. Hence the Reason why 60 Degrees is taken from the Line of Chords on any Scale for the Radius of a Circle suited to that Scale.

Theorem XVIII.

The Triangles form'd by the Radius Sine and Co-Sine of any Arch; as also by the Radius Tangent and Secant; and lastly, by the Radius Co-Tangent and Co-Secant of the same Arch, are all Similar, and have their Sides proportionable.

Demon-

Demonstration.



In the Three several Triangles ACD, FCE, GCH, ——

The several Angles of the one are equal to the several Angles of the other (by *Theo.* III.) thus,

D=E=H
ACD=FCD=CGH.
CAD=CFE=GCH.

Consequently the Sides about those equal Angles will be proportional (by *Theor.* XV) thus,

As AC : AD : : CF : FE : : GC : CH, as CD : AD : : CE : EF : : GH : HC. as CA : CD : : CF : CE : : CG : GH. *Q. E. D.*

The Proportions in Words.

1. As Radius is to the Sine of an Arch;
   So is the Secant to the Tangent thereof;
   So is the Co-Secant to the Radius.

2. As Co-Sine is to the Sine,
   So is Radius to the Tangent;
   So is the Co-Tangent to the Radius.

3. As Radius is to the Co-Sine,
   So is the Secant to the Radius;
   So is the Co-Secant to the Co-Tangent.

Corol.

Corol. I. Hence the Radius is a mean Proportional between the Sine and Co-Secant of an Arch. By Prop. 1. as AD : CA : : CH : GC.

Corol. II. The Radius is a mean Proportional between the Tangent and Co-Tangent of any Arch. By Prop. 2. as EF : CE : : HC : HG.

Corol. III. The Radius is a mean Proportional between the Secant and Co-Sine of an Arch. By Prop. 3. as CD : CA : : CE : CF.

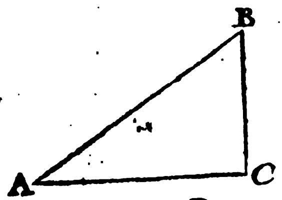
Corol. IV. Therefore in the Logarithmic Canon,
If from the Double of Radius,
You substract the Co-Sine of any Arch,
There will remain the Secant thereof.
If you substract the Sine of the Arch,
there will remain the Co-Secant.

Scholium.

This Excellent Theorem exhibits the Rational, or true Reason of all the Canons or Proportions made use of in all the Operations of Plain and Spherical Trigonometry. And the Reason why Secants are not in the Tables as well as Sines and Tangents, is because they are so easily found (when they are wanted) by *Corol.* IV.

Theorem XIX.

Any Two Sides of a Right-Angled Triangle being given, the other Side is also given.

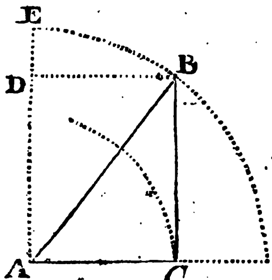


 Demon-

Demonstration.

For (by *Theor.* XI.) the Square of —— AB
is equal to the Sum of the Squares of AC and CB
Therefore the Square of the Side —— —— —— AC
is equal to the Dif. of the Squares of AB and CB
And the Square of the Side —— —— —— BC
is equal to Diff. of the Squares of —— AB and AC
Now since the Square of any Side —— AB, AC, BC
Is given; The Side it self is also given by Extraction. } *Q. E. D.*

Theorem XX.



In a Right-angled Triangle if the Hypothenuse be made Radius, then are the Sides the Sines of their opposite Angles; and if either Leg be made the Radius, then the other Leg is the Tangent of its opposite Angle, and the Hypothenuse is the Secant of that Angle.

Demonstration.

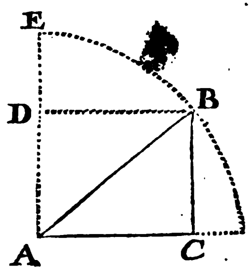
1. Make the Hypothenuse Radius, *viz.* —— AB
Then (by *Def.* XX.) the Side —— —— BC
Is the Sine of its opposite Angle —— —— BAC
And the Side or Leg —— —— —— AC=DB
is (by *Def.* XXIV.) the Sine of —— ABC=BAD
2. Again, if the Leg or Base be Radius —— AC
Then (by *Def.* XXI) the other Leg —— BC

Is

Is the Tangent of the Angle ——— BAC
And the Hypothenuſe ——— AB
Will be (by *Def.* XXII.) the Secant thereof. *Q.E.D.*

Theorem XXI.

The Radius and Sine of an Arch being given, the Co-ſine is alſo given.

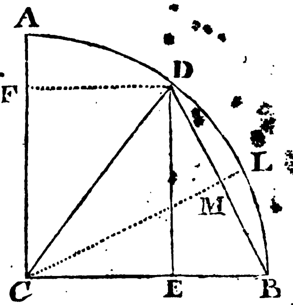


Demonſtration.

There being given the Radius and Sine——AB & BC,
Then (by *Theor.* XIX.) the Co-ſine—— AC - DB
is equal to the Square Root of —— ABq - BCq.

Theorem XXII.

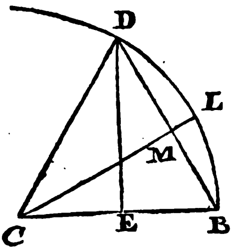
The Radius and Sine of any Arch being given, the Sine of half that Arch is alſo given.



Demonſtration.

There being given ——— CD (= CB) and DE,
There is given (by *Theo.* XXI.) ——— CE.

Then the Diff. of Co-sine and Radius, CB—CE,
Is the Side (or Versed-sine) —— —— —— EB,
Therefore in the Right-angled Triangle —— DEB
there is given the two Sides ————DE and EB
Whence (by *Theor.* XIX.) the other Side —— DB
is given also; The half of which, *viz.* —— DM
is the Sine of the Arch —— ——— —— DL
which is half the Arch ————DB. *Q.E.D.*



Theorem XXIII.

The Radius and Sine of an Arch being given, the Sine of double that Arch is also given.

Demonstration.

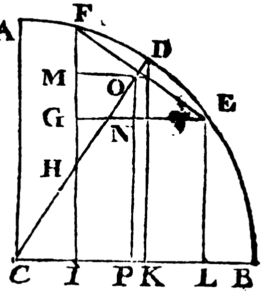
There being given the Radius and Sine, CB & BM
of the Arch —— —— —— BL,
there will be given (by *Theo.*XXI.) the Co-sine CM,
and because the Angles —— —— E=M
and the Common Angle —— —— B;
the Right-angled Triangle —— —— CBM
is Similar to the Triangle—— —— —— DEB.
Therefore (by *Theor.* XV.) As CB : CM : : DB (=2BM) : DE ———— } the Sine of double the given Arch, *viz.* of DLB.

Q.E.D.

Corol. I. Hence we have these Proportions,
As ———CB : 2CM : : BD : 2DE,
That is, as CB : 2CM : : 2BM : 2DE.
Therefore, as BM : DE : : $\frac{1}{2}$CB : CM.

Corol.

Corol. II. Wherefore, the Sine of any Arch and the Sine of its Double being given, the Co-ſine is alſo given, by the laſt Proportion.



Theorem XXIV.

The Sines of two Arches BD, FD, being given, the Sine of the Sum FI, and of the Difference EL, are both given likewiſe.

Demonſtration.

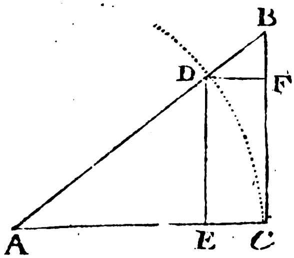
There being given the Radius and Sine CD, and FO.
the Co-ſine (by *Theo.* XXI.) is alſo given, *viz.* CO,
of the Arch ——— ——— ——— ——— FD.
And becauſe of the Parallel Lines ——— OP, DK,
and alſo of the Parallels ——— OM, GE, CB,
Thoſe Triangles will be ſimi-} CDK, COP, CHI,
lar, or Equiangular, *viz.* { FOH, and . FOM.
Therefore (by *Theor.* XV.) as CD : DK : : CO : OP.
which now is known; Alſo as CD : CK : : FO : FM,
which is alſo now known; but becauſe FO=EO,
Therefore the Lines ——— ——— FM=MG=ON
Hence it will be ——— ——— OP+FM=FI
The Sine Sum of the Arches; and alſo OP—FM=EL
The Sine of their Difference ——— ——— *Q. E. D.*

Corol. I. Hence becauſe the Differences of the Arches BE, BD, BF, are equal, BD will be an Arithmetical Mean between BE and BF.

Corol.

Corol. II. Hence Radius : to Double the Co-ſine of the Mean Arch : : the Sine of the Difference of the Arches : to the Difference of the Sines of the extream Arches.

For we have —— CD : CK : : FO : FM,
Conſequently, CD : 2CK : : FO : 2FM=FG,
The difference of the Sines EL, FI. *Q.E.D.*



Theorem XXV.

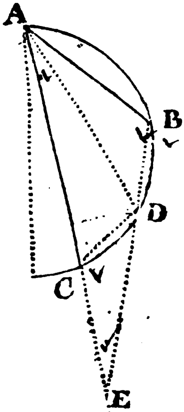
In ſmall Arches, the Sines and Tangents of the ſame Arches are nearly to one another in the Ratio of Equality.

Demonſtration.

Becauſe of the the Parallel Lines CB, DE,
Thoſe Triangles will be ſimilar DAE, BAC
Therefore (by *Theor.* XV.) as AE : AC
: : DE : BC. ———————————
But as the Point ——— ——— — E
Approaches to the Point ———— C
The Difference —— ——— —— EC
will vaniſh, in reſpect of the Arch DC
Whence the two Lines ——— AE, AC
will be nearly equal; ſo will DE, BC,
be nearly equal. If ——— —— EC
be leſs than the $\frac{1}{10.000.000}$ Part of AC
Then ſhall the Diff. of the Sine and Tan. BF
be leſs than the $\frac{1}{10.000,000}$ Part of——BC.
Therefore, &c. ——— ——— *Q. E. D.*

Corol.

Corol. Since any Arch is leſs than the Tangent, and greater than its Sine, and the Sine and Tangent of a very ſmall Arch are nearly equal, Therefore the Arch ſhall be nearly equal to its Sine; and ſo in very ſmall Arches it ſhall be, as Arch is to Arch, ſo is Sine to Sine.



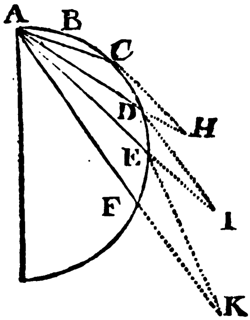
Theorem XXVI.

If the Angle BAC, being in the Periphery of a Circle, be biſected by the Right-Line AD; and if AC be produced until DE=AD meet it in E, then ſhall CE=AB.

Demonſtration.

In the Four-ſided Figure ——— ——— —— ABDC
The Angles (by *Theo.* XIII and L) —— B + ACD
are equal to two Right ones, and to DCE+DCA.
Whence by (*Ax.* V.) the Angle ——— B=DCE
Alſo (by *Theor.* VI.) the Angle ——— E=DAC
And (by Suppoſition) the Angle —— DAC=DAB
Alſo (by *Theor.* XII.) the Side ——— CD=DB
Wherefore the Triangles ——— ——— BAD & CED,
are equal (by *Theor.* VIII.) Therefore CE=AB
Q. E. D.

Theorem



Theorem XXVII.

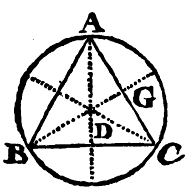
Let the Arches AB, BC, CD, DE, EF, *&c.* be equal; and let the Subtenses of the Arches AB, AC, AD, AE, *&c.* be drawn; then will AB : AC : : AC : AB + AD : : AD : AC + AE : : AE : AD + AF.

Since the Angle ——— ——— ——— BAD is bisected by the Right-Line ——— ——— AC we shall have (by *Theor.* XXVI.) ——— DH = AB So likewise shall ——— EI = AC, and FK = AD.

But the *Isosceles* Triangles ——— { ABC, ACH, ADI, AEK, are all Equiangular, by ——— *Theo.* XXVI.

Wherefore it will be (by *Theor.* XV. { As ——— ——— AB : AC : : AC : AH = AB + AD : : AD : AI = AC + AE : : AE : AK = AD + AF. *Q.E.D.*

Corol Because (by *Theor.* XXIII. *Corol.* 1.) AB : AC : : Radius : Double the Co-Sine of $\frac{1}{2}$ the Arch AB; Therefore it will be, As Radius : to Double the Co-Sine of $\frac{1}{2}$ the Arch AB : : $\frac{1}{2}$ AB : $\frac{1}{2}$AC : : $\frac{1}{2}$AC : $\frac{1}{2}$ AB + $\frac{1}{2}$AD : : $\frac{1}{2}$AD : $\frac{1}{2}$ AC + $\frac{1}{2}$AE : : $\frac{1}{2}$ AE : $\frac{1}{2}$ AD + $\frac{1}{2}$ AF, *&c.*



Theorem XXVIII.

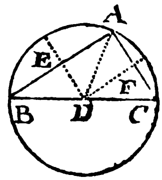
The Sides of Plain Triangles are to one another as the Sines of their opposite Angles.

Demon-

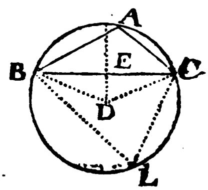
Demonstration.

1. In the Acute Angled Triangle —— ABC,
if a Side be bisected, as —— —— —— AC
then (by *Def.* XX.) Half the Side, viz. —— —— —— } AG, or CG
will be the Sine of the Angles ADG, or CDG,
and therefore (by *Theor.* XII.) of the opposite Angle, —— —— } ABC.

2. In the Right-Angled Triangle —— } ABC.
The Sine of the Right-Angle BAC is the Radius or ½ the Opposite Side —— } BD=½BC



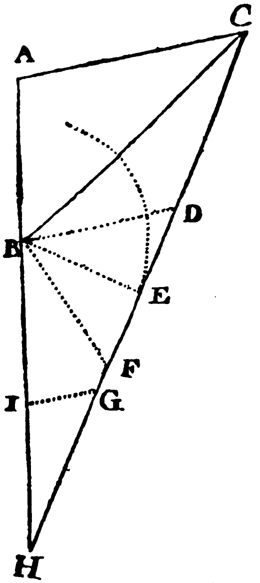
3. In the Oblique Angled Triangle —— —— } ABC,
The Angle —— —— —— BAC
hath the same Sine, as the Angle —— —— } BLC,



* (For they are either to other, the Complement to two Right Angles) by *Theor.* XIII.
But the Angle —— —— —— —— —— BDE,
(whose Sine is —— —— —— —— —— —— BE)
is (by *Theor.* XII.) equal to the Angle —— BLC
Therefore also shall —— —— —— —— —— BE
be the Sine of the Angle —— —— —— BAC.
And thus in every Triangle the Halves of the Sides are the Sines of their opposite Angles; but 'tis evident, the Sides are to one another as their Halves.
Ergo, &c. —— —— —— —— —— *Q. E. D.*

**N.B.* The Sine of an Arch and of its Complement to 180 Degrees, or two Right Angles, is all one.

F *Theorem*



Theorem XXIX.

In a Plain Triangle, As the Sum of the Legs, is to the Difference of the Legs; So is the Tangent of half the Sum of the opposite Angles to the Tangent of half their Difference.

Demonstration.

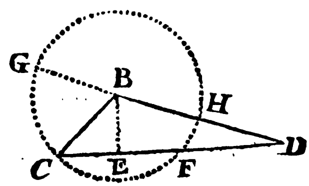
Let the given Triangle be —— —— ABC.
and let it be made BI=BA, BH=BC, HF = CD.
also let the Lines —— —— —— BD, IG, AC,
be parallel; and the Perpendicular Radius —— BE.
Then will the Line —— —— —— AH
be the Sum of the Legs —— —— AB+BC.
and their Difference will be —— —— IH.
Also the Sum of the opposite Angles A+ACB
will be (by *Theor.* V.) the Angle —— CBH,
the Half of which is the Angle —— —— CBE,
the Tangent whereof is —— —— —— EC.
And since (by *Theor.* VIII.) HBF=CBD=BCA
and (by *Theor.* III.) the Angle —— HBD=A.
Where-

Wherefore the Difference of the Angles A—ACB
shall be the Angle —— —— FBD,
whose Half is the Angle —— —— EBD,
the Tangent of which is —— —— ED.
But (by *Theor.* XV.) we shall have As HI : HG : : HB : HD
—— : : HA : HC.
Subduct the Terms or Proportionals, HI, HG,
the Remainders will be IB : BA : : GD : DC.
Whence we shall have —— —— DC=DG=HF
Consequently, —— HG=DF
and therefore —— —— $\frac{1}{2}$HG=$\frac{1}{2}$DF=DE.
And because of similar Triangles, AHC, IHG,
it shall be (by *The.* XV.) As AH : IH : : HC : HG
: : $\frac{1}{2}$HC : $\frac{1}{2}$HG : : EC : ED.
Therefore as —— ——AH : IH : : EC : ED.

Q. E. D.

Theorem XXX.

In any plain Triangle, as the Base or longest Side is to the Sum of the other two Sides; So is the Difference of those Sides, to the Difference of the Segments of the Base.



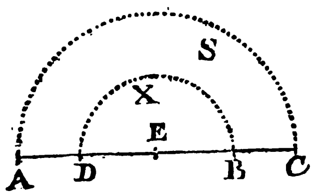
Demonstration.

In the Triangle —— —— —— CBD,
the Base or longest Side is —— —— —— CD.
The Sum of the other two Sides GD=CB+BD,
the Difference of which is —— —— HD,
The Segments of the Base are —— —— CE, ED,
whose Difference is —— —— —— FD.
And because (by *Theor.* XVI.) DC×DF=GD×DH,
Therefore 'twill be, As DC : GD : : DH : DF.

 That

That is, As the Base — — — DC
is to the Sum of the Sides — GD
So is their Difference — — DH
to the Diff. of the Seg. of the Base DF.
Q. E. D.

Theorem XXXI.



The Sum and Difference of any two Quantities being given, the Quantities themselves are thereby given.

Demonstration.

Suppose the two unknown Quantities are reperſented by the two Lines — } AB, BC.
Let the Sum of both be — — $AC=S$,
Then make — — — — $AD=BC$.
So ſhall their Difference be — $DB=X$.
Alſo the half Sum is — — $AE=\frac{1}{2}S$.
And the half Difference is — $EB=\frac{1}{2}X$.

But it is — — $AE+EB=\dfrac{S+X}{2}=AB$.

Alſo it is — — $AE-EB=\dfrac{S-X}{2}=BC$.

That is in Words,

Half the Sum added to half the Difference gives the greater Quantity AB; and half the Difference deducted from half the Sum, leaves the leſſer Quantity BC. *Q. E. D.*

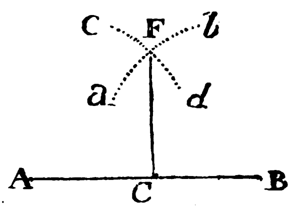
CHAP.

CHAP. IV.

Geometrical Problems.

Problem I.

UPON any given Point C, in a given Line AB, to raiſe a Perpendicular.

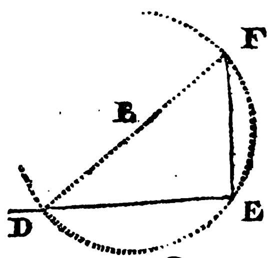


Practice.

On each Side the Point —— —— —— C,
take an equal Diſtance —— —— CA=CB,
then with one Foot of the Compaſſes in A and B
with any Diſtance greater than —— —— AC
deſcribe two interſecting Arks, —— —— a b, c d,
Join the two Points —— —— —— F, C,
And 'tis the Perpendicular required.

Problem II.

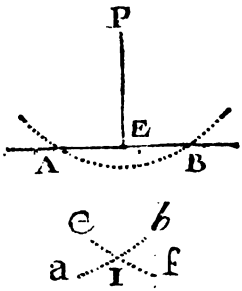
To raiſe a Perpendicular on the Point E, at the End of a given Line D E.



Practice.

Practice.

Set one Foot of the Compaſſes in ——— ——— E
and pitch the other in any Point ——— ——— B
above, or over, the given Line ——— ——— DE
with the ſame Diſtance on ——— ——— B
deſcribe the Semicircle ——— ——— ——— DEF
Draw the Diameter ——— ——— ——— ——— ——— DF
to cut the Semicircle in the Point ——— ——— F
join the two Points ——— ——— ——— ——— F, E;
And it is the Perpendicular requir'd.



Problem III.

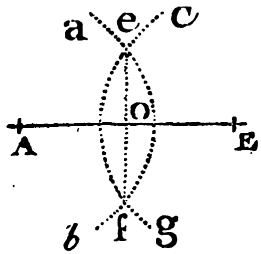
From a Point given above a Line as P, to let fall a Perpendicular on the Line AB.

Practice.

On the Point given ——— ——— ——— ——— P
with a Diſtance greater than ——— ——— PE
deſcribe an Arch, as ——— ——— ——— AB,
cutting the given Line in ——— A, and B;
On thoſe Points, deſcribe the two Arches, a b, e f,
then from the Point of Interſection ——— ——— I
lay a Ruler to the Point ——— ——— ——— P
and draw the Line ——— ——— ——— PE
That is the Perpendicular required.

Problem.

Problem IV.

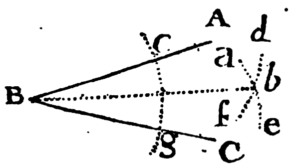


To divide a given Right-line, as A E, into two equal Parts, *i. e.* to bisect it.

Practice.

On the two Ends, — — — — A, E
with a sufficient Distance, describe — a b, c g,
then from the Points of Intersection, — — e, f,
draw the Line — — — — e f
cutting the given Line in — — — — O
and being just in the Middle, it divides the Line into the equal Parts — } AO=OE

Problem V.

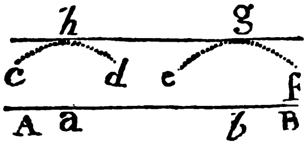


To divide a given Angle ABC into two equal Parts, by the Right-line B b.

Practice.

On the Angular Point — — — B
at proper Distance, describe the Arch, — c g
then on the Points — — — — c, and g,
describe the two intersecting Arches, a e, and d f,
join the angular and intersecting Points — B, b,
by the Right-line — — — — B b,
That Line divides the Angle into two equal Parts, as was required.

Problem

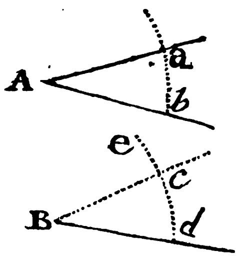


Problem VI.

To draw a Line parallel to a given Right-line at an any given Diſtance.

Practice.

In the given Line — — — — AB
on any two Points — — — — a b
with the given Diſtance deſcribe the two Arches — — — — } cd, ef,
on the Extreams of thoſe Arches — g, h
draw a Right-line, which will be parallel to the given Line, as required.



Problem VII.

To make a Right-lined Angle B, equal to a given Angle A.

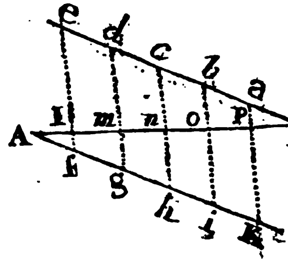
Practice.

On the given Angle — — — — A,
deſcribe an Arch, as — — — — a b,
Then, having drawn any Line — — Bd
on the Point — — — — — B
with the ſame Diſtance, as before, — — Ab
deſcribe the Arch — — — — — c d,
then ſet off — — — — — d c=a b
and through the Points — — — B, c,
draw the Right-line — — — — Bc
then ſhall the Angle — — CBd=aAb
as required.

Problem

Problem VIII.

To divide a Right-line into any Number of equal Parts; Suppose six.

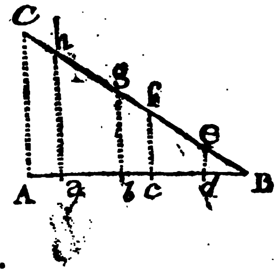


Practice.

On the Ends of the given Line, —— —— AB
and on contrary Sides, draw the Lines—— AK, Be,
parallel or at equal Angles; then on each make the equal Divisions (which must be but 5.) Ba, ab, bc, cd, de, Af, fg, gh, hi, iK,
then draw the Parallels ——— aK, bi, ch, dg, ef,
for they will divide the Line —— —— —— AB,
into Six equal Parts, in the Points —— l, m, n, o, p,
as was required.

Problem IX.

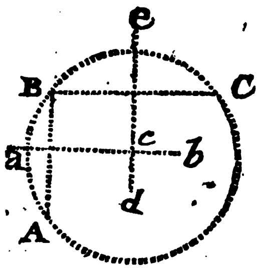
To divide a Right Line BC in the same Proportion as any other Line AB is divided.



Practice.

With the two Lines make any Angle —— ABC,
and suppose the Line —— —— AB
be divided in the several Points ——— a, b, c, d,

join the Points —— —— —— —— A, C,
to which draw the Parallels, —— ah, bg, cf, de,
these shall divide the Line————BC,
in the same Proportion in the Points, — e, f, g, h,
(by *Theor.* XV.) as was required.

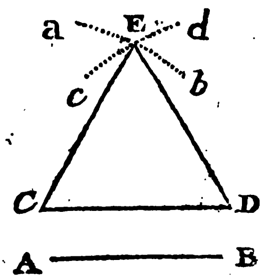


Problem X.

To describe a Circle that shall pass thro' any three given Points (not lying in a Right Line) as the Points A,B,C.

Practice.

Join the Points ———— ——— A, B, and B, C,
bisect those Lines, by the Sines —— a b and d e
whose Point of Intersection————— c
is the Center of the Circle passing thro' the three given Points, as was required. } A, B, C,



Problem XI.

To make an Equilateral Triangle whose Sides severally shall be equal to the given Line A B.

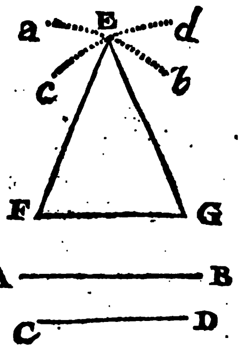
Practice.

Draw the Line —— —— —— CD=AB,
then on the Points———————C and D
and

and with the Diſtance ——— —— —— C D,
deſcribe the two Arches——— ——— ——— a b, c d
interſecting each other in ————————— E
Join the Points —— ——— ——— CE and DE
and the equilateral Triangle————————— CED
will be compleated, as required.

Problem XII.

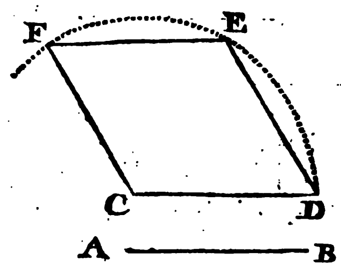
To deſcribe an Iſoceles Triangle, whoſe equal Legs ſhall be each equal to a given Line AB, and its other Side equal to another given Line CD.



Practice.

Make the Side ——— ——— ——— —— FG=CD,
and on the Points————————————— F, G,
and with the Diſtance of the Line ——— A B,
deſcribe the two Arches ————— a b, and c d,
interſecting each other in —— —— ——— E
Join the Points————————————FE, and EG,
then will the Iſoceles Triangle ———— FEG
be compleated, as required.

Problem XVI.

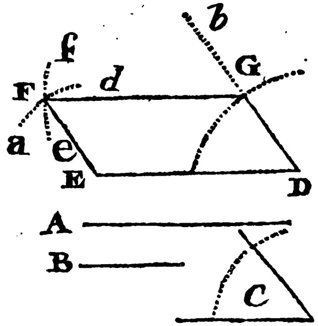


To conſtruct a Rhombus, whoſe Sides ſhall be equal to the given Line A B.

Practice.

Draw the Line —— —— —— CD=AB,
on the Central Point —— —— —— C
and with the Diſtance —— —— —— C D
deſcribe the Arch —— —— —— DEF
then from the Point —— —— —— D
ſet off on the Arch —— —— DE, EF,
each equal to the Line —— —— CD,
join the Points —— —— DE, EF, FC,
And the Rhombus is conſtructed, as required.

Problem XVII.



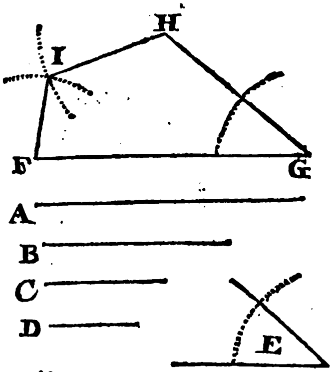
To conſtruct a Rhomboides, whoſe oppoſite Sides ſhall be ſeparately equal to two given Lines A, B; and whoſe Angle D ſhall be equal to the given Angle C.

Practice.

Practice.

Draw the Line ——— ——— ED=A.
then on the Point —— —— —— D
draw a Line ——— ——— D b
making the Angle —— —— D=C
then on the Line ——— ——— D b
set off the Side ——— ——— DG=B
On the Points ——— ——— G, E,
and with the Distances —— ——— ED, DG,
describe the two Arches ——— —— a d, e f,
and from the Point of Intersection —— F
draw the Lines, or Sides ——— —— GF, FE
And the Rhomboides is compleated, as required.

Problem XVIII.

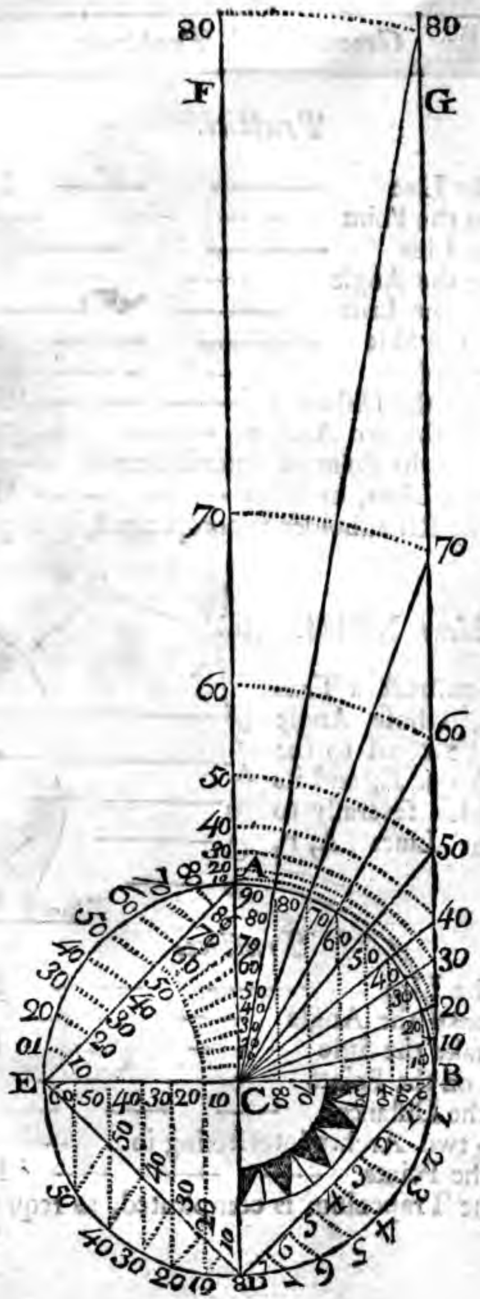


To construct a Trapezium, whose Angle G shall be equal to the given Angle E, and its four Sides severally to the four Lines A, B, C, D.

Practice.

Draw the Line ——— ——— FG=A
and make the Angle —— —— G=E
also make the Side —— ——— GH=B
Then on the Points —— —— H, F
with the Distances —— —— —— C, D,
sweep two Arches intersecting in ——— I
Join the Points —— —— —— IH, IF,
And the Trapezium is compleated, as required.

Problem



80
80
F
G
70
70
60
60
50
50
40
40
A
E
C
B
D

Problem XIX.

To construct a Scale of Natural Sines, Tangents, and Secants; also a Line of Chords, Rhumbs, and Longitudes.

Practice.

Describe a Circle ——— ——— ABDE
which cross with two Diameters ——— AD, BE,
on the Point ——— ——— B
erect the Perpendicular ——— ——— BG,
parallel to which continue the Diameter ——— DA
infinitely towards ——— ——— F
then graduate the quadrantal Arch ——— AB
and thro' each Division, from the Center ——— C
draw Lines to the Perpendicular ——— BG
which Perpendicular shall be the Tangent-Line, graduated in the Points. ——— 10. 20. 30. 40. &c.
Then with one Foot of the Compasses in ——— C
transfer the Divisions of the Tangent ——— BG
with the other Point to the Line ——— AF
so shall that be a Line of Secants, graduated in the Points ——— ——— 10. 20. 30. 40. &c.
Then from the said Divisions of the Arch ——— AB
let fall Perpendiculars to the Radius ——— CB
so shall that Radius be a Line of Right-Sines, graduated in the Points. 10. 20. 30. &c.
Also, if from the Point ——— ——— E
you draw Lines to the Divisions in ——— AB
they will graduate the Line ——— ——— AC
for a Line of half Tangents, in 10. 20. 30. 40. &c.

H Next

Next graduate the Arch —— —— AE
and transfer thoſe Diviſions to the Chord —— AE
ſo ſhall that be a Line of Chords graduated in the Points —— —— } 10. 20. 30. 40. &c.
Divide the Semidiameter —— —— CE
into 60 equal Parts, in —— —— 10. 20. 30. &c.
from which drop Perpendiculars to the Arch ED
transfer thoſe Diviſions in the Arch, to the Chord —— —— — } ED
ſo ſhall that Chord be the Line of Longitudes, graduated in the Points —— —— } 10. 20. 30. &c.
Laſtly, divide the quadrantal Arch —— BD
into Eight equal Parts, in —— 1. 2. 3. 4. &c.
then transfer thoſe Diviſions to Chord —— BD
ſo ſhall that Chord be a Line of Rhumbs, graduated in the Points } 1. 2. 3. 4. &c.

Thoſe Lines being thus made and graduated, are to be placed appoſitely on any Inſtrument of Paſte-board, Wood, or Braſs, &c. and this is what is commonly called the *Plain-Scale*; which may be made of any Size according to the Radius or Semidiameter of the Circle.

N.B. This *Problem* ought to be well underſtood by every one who would have a rational Knowledge of *Trigonometry*, the Conſtruction of this Scheme being the Grounds and Rationale of every Kind thereof.

CHAP.

CHAP. V.

Various Methods of Constructing a Canon *of* Natural Sines, Tangents, *and* Secants : *Also of the* Logarithmetic Canon.

THE first Method for making Natural Sines, *&c.* is deduced from the foregoing *Theorems.*

For, by *Theorem* XVII, the Chord of 60 Degrees is equal to the Radius or Semidiameter of the Circle; consequently, half the Radius is the Sine of 30 Degrees; and therefore if Radius be supposed equal to 10000000 Parts, the Sine of 30 Degrees shall be equal to 5000000 of the said Parts.

Now, the Sine of the Arch of 30 Degrees being given, the Sine of the Arch of 15 Degrees is given also (or may be found) by *Theorem* XXII; By the same *Theorem* also, the Sine of 15 Degrees being given, we shall have the Sine of 7 Deg. and 30 Min. and this Sine being known, we shall find the Sine of 3 Deg. 45 Min. by the same *Theorem*; and thus proceeding, till 12 Bisections being made, we come to the Sine of an Arch of 52'' 44''' 03'''' 45''''' whose Co-Sine is nearly equal to Radius.

In such a Case, 'tis evident by *Theorem* XXV, that Arches are proportional to their Sines; and

 therefore,

therefore, As the Arch of 52″ 44‴ 03⁗ 45′′′′′ is to an Arch of one Minute; So is the Sine of the Arch of 52″ 44‴ 03⁗ 45′′′′′ to the Sine of an Arch of one Minute; which therefore is thus known.

The Sine of an Arch of one Minute being found, we find, by *Theorem* XXI, its Co-Sine, or Sine of 89° 59′, and the Sine of an Arch of two Minutes will become known, by *Theorem* XXIII. and its Co-Sine also (or Sine of 89° 58′) as beforesaid.

The Sines of the Arches of 1′ and 2′, as also their Co-Sines, *viz* the Sines of 89° 59′ and 89° 58′ being thus found; if, in the Scheme to *Theo.* XXVII. we make each of the Arches AB, BC, CD, *&c.* equal to 2′; then $\frac{1}{2}$AB=1′; and $\frac{1}{2}$AC=2′; and $\frac{1}{2}$AD=3′; *&c.* and let the Co-Sine of one Minute, *viz.* 89° 59′ be called K; then the Sines of all the other Minutes of the Quadrant may be found by *Corol.* to the said *Theorem* XXVII, and *Corol.* 1. to *Theorem* XXIII.

Thus the Proportion will be; As R : 2K : : Sine of 2′ : Sine of 1′ + Sine of 3′; thus the Sine of 3′ is found. Also R : 2K : : S. 3′ : S. 2′ + S. 4′; hence the Sine of 4′ is found. Again, As R : 2K : : S. 4′ : S. 3′ + S. 5′; wherefore the Sine of 5′ is thus known. Also, As R : 2K : : S. 5′ : S. 4′ + S. 6′; and so the Sine of 6′ will be had, and in like manner will all the other Sines be obtain'd.

But, because the Radius is to Double Co-Sine of an Arch of one Minute, as 1 is to 2; (for the Sine of 89° 59′ is equal to Radius, to a Number of Figures exceeding the Tables) therefore the above Proportions will stand thus,

As 1 : 2 : : S. 2′ : S. 1′ + S. 3′; As 1 : 2 : : S. 3′ : S. 2′ + S. 4′, *&c.* That is, if from the Double of any given Sine, be subducted the Sine of the Minute next before, there will remain the Sine of the Minute next following.

For

For Example, Suppose the Sine of 6° 1′ were required (all the precedent Sines being found) the Proportion will be, As 1 : 2 : : S. 360′ : S. 359′ + S. 361′.

Therefore multiply the Sine of 360′	=	1045285
By	——— ———	2
From that Product	—— ——	2090570
Subduct the Sine of 359 (=5° 59′)	=	1042392
There remains the sought Sine of 361′	=	1048178

And thus by an easy Multiplication and Subtraction all the Sines are found for every Minute of the Quadrant.

If the Sines and Co-Sines of 1′ and 2′ be given; the Sines of the following Minutes may also be found by the Proportion in *Corol.* II. to *Theorem* XXIV. For thereby 'twill be, As Radius : to Double the Co-Sine of 2′ : : Sine of 1′ : the Difference of the Sines of 1′ and 3′ : : Sine 2′ : the Difference of the Sines of 0′ and 4′, that is, to the Sine of 4′. And thus the Sines of the four first Minutes being given, we can thereby find the Sines of the others to 8′, and from thence to 16′, and so on till you arrive to Degrees.

By the same Proportion, the Sines of all the precedent Degrees being given to any Arch, we find the Sine of all subsequent Degrees to the Double of that Arch; As suppose, for Example, all the Sines to 15 Degrees be given; then, by the said Analogy, all the Sines to 30 Degrees may be found. For Radius : Double the Co-Sine of 15° : : Sine 1° : Difference of the Sines of 14° and 16° : : S. 2° : Difference of the Sines of 13° and 17° : : S. 3° : the Difference

ference of the Sines of 12° and 18°, &c. to the Sine of 30°. Then, all the Sines to 30° being given, As Radius : to Double the Co-Sine of 30° (= Double the Sine of 60°) : : S. 1° : Difference of the Sines of 29° and 31° : : S. 2° : Diff. of the Sines of 28° and 32°, &c. till you come to the Sine of 60°. But, in this Case, As Radius is to Double the Co-Sine of 30°, so is 1 to $\sqrt{3} = 1.7320508$, &c. and therefore if the Sines of the Distances from the Mean Arch of 30° be multiplied by 1.7320508 the Differences of the Sines will be obtain'd ; Since if Radius be 1, the Double of the Co-Sine of 30° (or Sine of 60°) will be equal to $\sqrt{3} = 1.7320508$, &c. as before.

Hence the Sines of all Arches from the Beginning of the Quadrant to 60 Degrees being given, distant from each other by a given Interval ; all the other Sines to 90° are easily found by one Addition only. For, in the Scheme to *Theorem* XXIV. if the Arch BD be equal to 60° the Difference of the Sines FI and EL, shall be equal to the Sine FO of the Distance FD from the Mean Arch of 60° BD, that is FO= FG ; because CK = Sine of 30°, and therefore, CD being equal to 2CK, by Similarity of Triangles we shall have FO = $\frac{1}{2}$FH=FG, as was said. Now, if the Arches DF, DE, be each equal to one Degree ; then shall the Arch BE=59°, and the Arch BF= 61°. But the Sine FI of the Arch BF=61°, is equal to the Sine EL of the Arch BE = 59° + the Sine FO (=FG) of the Arch FD=1°, and therefore the Sine 62°= S. 58° + S. 2° ; And the S. 63° = S. 57° + S. 3° ; and the S. 64° = S. 56°+S. 4° ; &c. so on, till, by this continual Addition, all the Sines be compleated.

Having by these Methods constructed a Canon or Table.

Then

of natural Sines for every Degree and Minute of the Quadrant, we may easily find the Tangents and Secants by the Proportions in *Theorem* XVIII.

Thus, for the Tangents, say; As the Co-Sine is to the Sine, so is the Radius to the Tangent;

And, As Co-Sine is to Radius, so is Radius to the Secant; that is, Divide the Square of Radius by the Co-Sine, the Quotient is the Secant. But both the Tangents and Secants will be best and easiest found by the Logarithms of the Sines, as I shall shew by and by.

The next, which is also the best and most absolute Method of finding the Natural Sines and Co-Sines of all Arches, is by a converging Series. The first that laid down a Series converging *in Infinitum*, for this Purpose, was that incomparable Mathematical Philosopher, Sir *Isaac Newton*, and after him it has been improved by others. The Genesis, or Method of Forming such a Series, may be found in several Authors, particularly, in *Domckius's Philos. Mathem. Newton. Illustrata.*

By this Method of converging Series the Arches must be given, in order to find the Sines thereof, and therefore the young Geometer must know, that if the Diameter of a Circle be 2, the Periphery, or whole Arch of such a Circle has been found (by very great Pains and Industry of the learned) to be, in that Measure, 6.28318530717958647692528676655900576. But 6 or 7 of the first of these Decimals are sufficient for Use. If then the whole Circumference of a Circle be 6.2831853, *&c.* one fourth Part, *viz.* 1.5707963, *&c.* will be the Length of the Arch of a Quadrant; and because every Quadrant contains 90 Degrees; therefore a Ninetieth Part of the last Number, *viz.*

0.0174533,

0.0174533, &c. is the length of an Arch of one Degree, and a Sixtieth of that, *viz.* 0.000290888, &c. is the Measure of an Arch of one Minute, and by such kind of Division the Length or Measure of an Arch of any Number of Degrees and Minutes, is to be found.

In this Case then, the Radius (or Semidiameter) being 1, and the Arch being found as before, call it A; and the Sine thereof will be

$$A - \frac{A^3}{1.2.3.} + \frac{A^5}{1.2.3.4.5.} - \frac{A^7}{1.2.3.4.5.6.7.} +$$

$$\frac{A^9}{1.2.3.4.5.6.7.8.9.}$$ &c. The Co-Sine will be

$$1 - \frac{A^2}{1.2.} + \frac{A^4}{1.2.3.4.} - \frac{A^6}{1.2.3.4.5.6} + \frac{A^8}{1.2.3.4.5.6.7.8.}$$

&c. The Tangent will be

$$A + \frac{A^3}{3} + \frac{2A^5}{15} + \frac{17A^7}{315} + \frac{62A^9}{2835}$$ &c.

These Series in the Beginning of the Quadrant, when the Arch A is but small, soon converge. For in the Series for the Sine, if the Arch does not exceed 10 Minutes, the two first Terms thereof, *viz.* $A - \frac{1}{6}A^3$ gives the Sine to 15 Places of Figures; if the Arch A be not greater than one Degree, the three first Terms will exhibit the Sine to 15 Places of Figures: Yea, the two first will give the Sine as far as the Tables require.

Example.

Example.

The Len. of the Arch of 1 Deg. is 0.01745329 = A.
The Logarithm whereof is ——— .8 2418772
Multiply that by ——— ——— 3

The Product is the Cube, *viz.* A^3 .4.7256316
Which divide by —— ——— 6 = 0.7781512

The Quotient is $\frac{1}{6}A^3$ = 0.0000008861 = .3.9474804

Then from the first Term—— A = 0.017453292 &c.
Substract the second Term $\frac{1}{6}A^3$ = 0.000000886

There remains the Sine of 1 deg. = 0.017452406
the same as in the Tables.

Example of the Co-Sine of one Degree.

The Logarithm of A, as before, = .8.2418772
Which multiply by ——— 2

The Product is the Log. of A^2 = .6.4837544
From which substract the Log. of 2 = 0.3010300

There remains $\frac{A^2}{1.2.}$ = .0001523 = .6.1827244

Now from the first Term, *viz.* Unity 1.0000000

Substract the second Term— $\frac{A^2}{1.2.}$ = 0.0001523

There remains the Co-S. of 1 Deg. = 0.9998477
the same as in the Tables.

Example of the Tangent of one Degree.

The Log. of A^3, as before, is —— .4.7256316
From which substract the Log. of 3 = 0.4771212

There remains $\frac{A^3}{3}$ = 0.000001772 = .4.2485104

Therefore to the first Term A = 0.017453292 &c.

Add the second Term —— $\frac{A^3}{3}$ = 0.000001772

The Sum is the Tang. of 1 Deg = 0.017455064

Hence 'tis manifest, that only the two first Terms in either of the Series are necessary for finding the Sines, Co-Sines, and Tangents, to 6 or 7 Places (which is as far as the Tables go) for all Arches not exceeding one Degree. But when the Arch is greater than one Degree, more Terms of the Series must be used. So that 4 Terms of the Series for Sines are necessary to find the Sine of 25 Degrees, true to but 6 Places of Figures. As will appear by the following Operation.

The Length of an Arch of 25 deg. when the Radius is an Unit — is } 0.436332226 &c.
The Logarithm of which is ——— .9.6398172
Multiply that by——— —— ——— 3

The Product is the Log. of A^3 = .8.9194516
From which substract the Log. of 6 = 0.7781512

Rem. the 2d Ter. $\frac{A^3}{1.2.3.}$ = 0.01384523 = .8.1413004

Again,

Again, the Logarithm of A^5 is —— .8.1990860
From which subst. the Log. of 120 = 2.0791812

Rem. the 3d Ter. $\frac{A^5}{120} = 0.00013179 = .6.1199048$

Lastly; the Logarithm of A^7 is .7.4787204
From which subst. the Log. of 5040 = 3.7024305

Re. the 4th Ter. $\frac{A^7}{5040} = 0.0000005974 = .3.7762899$

Hence, if from the first Term $A = 0.43633226$

we subduct the second Term $\frac{A^3}{6} = 0.01384523$

and to the Remainder $A - \frac{A^3}{6} = 0.42248703$

we add the third Term —— $\frac{A^5}{120} = 0.00013179$

and again from the Sum $A - \frac{A^3}{6} + \frac{A^5}{120} = 0.42261882$

we substract the fourth Term $\frac{A^7}{5040} = 0.00000059$

The Remains the Sine of 25 Deg. $= 0.42261823$

But this *Newtonian* Series converging so very slowly when the Arch is of any considerable Length, and the more slowly, the greater the Arch is; therefore the late learned Dr. *John Keil* devised other Series, similar to Sir *Isaac's*, in order to remedy the Deficiencies of that; and in those Series of his

composing, he supposeth the Arch, whose Sine is sought, is the Sum or Difference of two known Arches, *viz.* A, and z; that is, A+z, or A−z. And if the Sine of the Arch A be called a, and the Co-Sine b; then the Sine of the Arch A+z will be thus expressed;

$$1.\ a + \frac{bz}{1} - \frac{az^2}{1.2.} - \frac{bz^3}{1.2.3.} + \frac{az^4}{1.2.3.4.} + \frac{bz^5}{1.2.3.4.5.}$$

$$- \frac{bz^6}{1.2.3.4.5.6.}$$ &c. And the Co-Sine will be,

$$2.\ b - \frac{az}{1} - \frac{bz^2}{1.2.} + \frac{az^3}{1.2.3.} + \frac{bz^4}{1.2.3.4.} - \frac{az^5}{1.2.3.4.5.}$$

$$- \frac{bz^6}{1.2.3.4.5.6.}$$ &c. In like Manner the Sine of the Arch A−z is

$$3.\ a - \frac{bz}{1} - \frac{az^2}{1.2.} + \frac{bz^3}{1.2.3.} + \frac{az^4}{1.2.3.4.} - \frac{bz^5}{1.2.3.4.5.}$$

$$- \frac{az^6}{1.2.3.4.5.6.}$$ &c. And the Co-Sine thereof is

$$4.\ b + \frac{az}{1} - \frac{bz^2}{1.2.} - \frac{az^3}{1.2.3.} + \frac{bz^4}{1.2.3.4.} + \frac{az^5}{1.2.3.4.5.}$$

$$- \frac{bz^6}{1.2.3.4.5.6.}$$ &c.

Now,

Now, as all these Series easily flow from the *Newtonian* ones, so they may as easily be illustrated by *Theorem* XXIV. for, if (in the Scheme thereto) we put the Arch BD$=$A, and the Arch DF, or DE $=z$; then shall the Arch BF$=$A$+z$; and the Arch BE$=$A$-z$; and in the first and second Series, we have $a=$DK, and $b=$CK; and the whole first Series will be equal to FI (*viz.* the Sine of A$+z$;) and the whole second Series will be equal to CI (the Co-Sine of A$+z$.) Also in the third and fourth Series, we have a and b the same as before, and the whole third Series $=$EL; and the whole fourth Series $=$CL. And the Arch A is an Arithmetical Mean between the Arches A$+z$ and A$+z$, by *Corol.* 1. to that *Theorem.* And the Difference of the Sines of the Arches A$+z$ and A, is

$$5.\quad \frac{bz}{1}-\frac{az^2}{1.2.}-\frac{bz^3}{1.2.3.}+\frac{az^4}{1.2.3.4.}+\frac{bz^5}{1.2.3.4.5.}$$

$$-\frac{az^6}{1.2.3.4.5.6.}\ \&c.$$ Also the Difference of the Arches A and A$-z$, is

$$6.\quad \frac{bz}{1}+\frac{az^2}{1.2.}-\frac{bz^3}{1.2.3.}-\frac{az^4}{1.2.3.4.}+\frac{bz^5}{1.2.3.4.5.}$$

$$+\frac{az^6}{1.2.3.4.5.6.}\ \&c.$$ Whence the Sum of those Differences, is

Hh2 bx

$$7.\quad 2b \times \frac{z}{1} - \frac{z^3}{1.2.3.} + \frac{z^5}{1.2.3.4.5.} \text{ \&c.} = FO_2 \times 2CK.$$

And the Difference of the Differences, or ſecond Difference, is

$$8.\quad 2a \times \frac{z^2}{1.2.} - \frac{z^4}{1.2.3.4.} + \frac{z^6}{1.2.3.4.5.6.} \text{ \&c.} = 2DK$$

$\times$ DO; that is, equal to Double the Sine of the Mean Arch multiplied into a Verſed Sine of the Arch z.

This ſecond Difference therefore is found by a very eaſy Operation; and having that, 'twill be as eaſy to find the Sines of all Arches diſtant from each other by equal Intervals.

But by the ſeventh Series, which is equal to Double of the Co-Sine of the Mean Arch drawn into the Sine of the Arch z, is the moſt eaſy and ready Method for finding the Arch $A+z$, or BF. The Application of theſe infinite Series is evident by a due Conſideration of the aforeſaid Scheme, let the ſeveral Arches be what Lengths they will, provided they be in Arithmetical Proportion.

The famous Mr. *Ward*, in his *Mathematician's Guide*, has propoſed two Biquadratic adfected Equations, by which the Sine of an Arch may be found, without the Help of any precedent Sine. But I conceive, were the Reader to compare the Operations by the foregoing Series, and by the Equations he has propoſed, the Preference muſt needs be given to the former, as abundantly the moſt natural, ſimple, and eaſy, as well as expeditious; and therefore I have not inſerted them here: For, I purpoſe to impoſe on the young Student nothing that is not of the greateſt Conſequence, and very neceſſary for him to be acquainted withal.

But

But, tho' I have already ſhewn how, by having both the Sine and Co-Sine of any Arch given, to find the Tangent of that Arch. I ſhall now ſhew how, by two *Theorems*, to find the Tangent, when only the Sine or the Co-Sine is given.

In order to that, in Scheme to *Theorem* XXV. let Radius AD$=1$; the Sine DE$=a$; the Co-Sine AE$=b$; and the Tangent CB$=$T. Then by an eaſy Algebraic Proceſs, we ſhall have the two following *Theorems*, *viz.*

If the Sine be given, then $\sqrt{\frac{aa}{1-aa}}=\text{T}.$

But if the Co-ſine be given, then $\sqrt{\frac{1-bb}{bb}}=\text{T}.$

That is in Words, The Square Root of the Square of the Sine, divided by Unity, leſs the Square of the Sine, is equal to the Tangent. Or, the Square Root of Unity, leſs the Square of the Co-Sine, divided by the Square of the Co-Sine, is alſo equal to the Tangent.

By the Methods above deliver'd, a Table or Canon of Natural Sines, Tangents, and Secants for the Arches of every Minute and Degree of the Quadrant, is readily made or conſtructed. And farther, there is no Occaſion to proceed; ſince a Quadrant contains all the Sines, Tangents and Secants, that can be form'd for any Arch of the Circle; for, as 'tis evident, we cannot ſpeak of the Sine of an Arch greater than 180 Degrees, or a Semicircle, ſo 'tis as plain, that the Sine of any given Arch, is alſo the Sine of that Arches Complement to 180 Degrees; thus the Sine of 28 Deg. is alſo the Sine of 152 Deg. the Reaſon of which is obvious from the Definition

of

of thofe Lines, and the Schemes thereto. And hence the Ambiguities attending the Solution of fome Cafes of Oblique Angled Plain Triangles; the Means of refolving which fhall be taught in due Place.

Having a Table of Natural Sines, Tangents and Secants, conftructed as has been taught, ready at Hand; we may thereby refolve all the Cafes of Plain and Spherical Trigonometry, and feveral have taught the Art in no other Way. But as the Operations by Natural Numbers (which confift of feveral Places of Figures) were very operofe and tedious, being large Multiplications and Divifions, fo the Wit, Invention and Induftry of Artifts was undoubtedly often exercifed to find out fome other more eafy and expeditious Methods of performing fo general, neceffary and ufeful a Part of Art. And at length the grand *Defideratum* appear'd, exceeding all their Wifhes, in the Difcovery of an Art (or rather, as I may call it, a Numerical Kind of Magick) noble as its Author, the wonderful Art of Logarithms, invented at firft by the Lord *Neiper*, Baron of *Merchifton* in *Scotland*, and firft publifhed at *Edenburgh*, in the Year 1614.

By this moft excellent Art, the elaborate Operations of Multiplication and Divifion are perform'd by Addition and Subftraction only of two Logarithmic Numbers, and confequently the whole Bufinefs of Trigonometry is wonderfully facilitated and expedited thereby. The Logarithms therefore of the Numbers expreffing the Natural Sines, Tangents and Secants, being difpofed into Tables, are called Artificial Sines, Tangents, &c. and with them we work now-a-days inftead of the other. But, if any be not acquainted with the Ufe of Logarithms, they may compleatly learn it, after the beft Manner, in my new Syftem of *Decimal Arithmetic*.

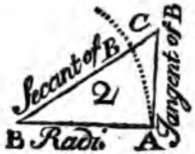
CHAP.

CHAP. VI.

Of the Solution of the Six Cases of Right-angled Plain Triangles.

HAVING in the precedent Theorems fully exhibited the Rationale, or true Reaſon and Foundation of the whole Doctrine of *Plain Trigonometry*, and the Manner of Forming Analogies, or Proportions for practical Calculations; I ſhall here preſent the Reader with a Synopſis, or General Scheme of the whole Doctrine of Right-angled Plain Triangles in one View; And according to what is given, and what Side is made Radius in any Triangle, the ſeveral Proportions are exhibited for finding the *Quæſita* in particular Caſes.

The Caſes of Right-angled Plain Triangles are in Number Six: For, ſince all the Parts of a Triangle are but Six, *viz.* three Sides and three Angles; and one of theſe, *viz.* the Right-Angle, is always known; there can be but five of thoſe Parts unknown; and two of thoſe being the two Acute Angles, which together are always equal to a Right Angle, or 90 Degrees, if one of them be known, the other muſt of Neceſſity, therefore both thoſe Acute Angles can be reckoned no more than as one *Datum*, or *Quæſitum*, which, with the three Sides, make but four Things in the Triangle unknown, and any two of which is ſufficient (with the Right-Angle) to find the Reſt. But the Combination of two Quantities in Four, are but Six. Therefore the Caſes of Right-angled Trigonometry are but Six, which follow.



The Six Cases.	Parts given.	Parts sought.			
I.	A B, B, C	A C, B C.	sC : AB : : sB : AC. sB : AC : : R : BC.	R : AB : : tB : AC. R : AB : : seB : BC	tC : AB : : R : AC. R : AC : : seC : BC.
II.	A C, B, C.	A B, B C.	sB : AC : : sC : AB. sB : AC : : R : BC.	tB : AC : : R : AB. R : AB : : seB : BC.	R : AC : : tC : AB. R : AC : : seC : BC.
III.	B C, B, C	A B, A C.	R : BC : : sC : AB. R : BC : : sB : AC.	seB : BC : : R : AB. R : AB : : tB : AC	seC : BC : : tC : AB. tC : AB : : R : AC
IV.	A B, A C.	B C, B, C.	*Analogiis caret.*	AB : R : : AC : tB. R : AB : : seB : BC.	AC : R : : AB : tC. R : AC : : seC : BC
V.	A B, B C.	A C, B C.	BC : R : : AC : sC. R : BC : : sB : AC.	AB : R : : BC : seB. R : AB : : tB : AC	*Analogiis caret.*
VI.	A C, B C.	A B, B, C.	BC : R : : AC : sB. R : BC : : sC : AB.	*Analogiis caret.*	AC : R : : BC : seC. R : AC : : tC : AB

In

In this Synopsis tis manifest that when two Sides of a Triangle are given, tis necessary to make one of them Radius in order to work by Proportions, or solve the Case by Sines, Tangents, and Secants; as appears in the IV Case of the first Triangle, the V Case of the 3d, and VI Case of the 2d Triangle, where there is wrote, *Analogiis caret*, *i. e.* It wanteth Proportions.

Now to the Solution of the foregoing Cases, in Order to which divers Methods may be used, as Occasion or Necessity shall require. Some of which are most General, extending to all the Cases of all kind of Triangles, others are particular; some are most exact and accurate, others more uncertain, and near the Truth only: Some are performed by Numbers only, some by Numbers and Species; and others by Instruments of several Kinds. The principal Methods for resolving the several Cases of Triangles are those which follow:

Method I. By Logararithmic Sines, Tangents, and Secants.

Method II. By Natural Sines, Tangents and Secants.

Method III. By the Trigonometrical sliding Scale.

Method IV. By Gunters Trigonometrical Scale, and Compasses.

Method V. By the Sector.

Method VI. By Geometrical Construction.

Method VII. and VIII. By the Practical Trigon, and Sinical Quadrant.

Method IX. By Natural Arithmetic.

Method X. By Algebra, or Analytical Investigation.

Of all those Methods, the First is most accurate, easy, and expeditious; and should be always used by the Young Trigonometer, when ever they can. It will not, I presume, be unacceptable to those Young and Ingenious Students who desires a thorough Knowledge in this most comprehensive and useful Part (or

rather, Summary) of the Mathematicks, to have those several Methods above-mention'd exemplified in such Cases wherein each Method is more particularly necessary; and this will be, to the apprehensive Youth, as sufficient, as if every Case in every Triangle were treated in every particular Method it was capable of; which would of it self make a large Volume.

In order that the Young Trigonometer may proceed with Clearness and Certainty, I shall lay down the following Precepts, which he must carefully observe in his Operations, especially those which are wrote by Analogy.

Precept I. In forming you Analogies, you must always compare opposite Sides to opposite Angles; and *contra*.

Precept II. When a Side is required, you must begin with an Angle; and when an Angle is sought, begin with a Side.

Precept III. When the Hypothenuse is given, you must work with the Sine or Co-Sine according as the Side sought, is opposite or adjacent to the given Angle.

Precept IV. When the Hypothenuse is not known, you must work with Tangents and Co-Tangents, or with Secants; according as the Side is opposite or adjacent to the Angle.

Precept V. Consider, that there is in the Tables of Sines, Tangents, &c. a Triangle exactly similar to the Triangle you are to solve; and whose Sides expressed in the Tables, are in the very same Proportion as those of the Triangle proposed.

Precept VI. Therefore (by *Theorem* XV.) you must say, As the Length of any one Side, in Inches, Yards, Miles, &c. of the tabular Triangle is to a similar Side of the same Measure in your Triangle; So is any other Side of the tabular Triangle, to the similar

similar Side sought in your given Triangle; which Sides must be properly expressed by *Precept* 3. 4.

Precept VII. When you are to use the Tables of Logarithms, if it be for the Logarithm of a Common Number, which in the Tables proceed from 1 to 1000, look in the Column under the Letter N, for the given Number, and right against it in the Column under the Word *Logarith.* you find its Logarithm: Thus, against the natural Number 2165 you find its Logarithm, 3.3354579.

Precept VIII. If you are to find the Logarithm of any Sine, Tangent, or Secant of any Angle; seek, in the Canon of Artificial Sines, Tangents, &c. for the Degrees on the Top of the Page, and the Minutes in the first Left-hand Column downward under the Letter M; if the Degrees be under 45°. thus against 25° 47′ you find the Logarithm of the Sine (under the Word *Log. Sinus*) to be 9.6384585, and the Tangent (under the Word *Log. Tang.*) to be 9.6840011.

Precept IX. If your Degrees be more than 45°, you must seek the Degrees at the Bottom of the Page, and the Minutes in the Right-hand Column upwards; and thus against 57° 35′ you find the Sine (under *Log. Sine*) to be 9.9264310; and the Tangent (under *Log. Tang.*) to be 10.1972075. After the same manner, the Natural Sines, Tangents, &c. are to be found.

Precept X. If the Table of Logarithmic Sines and Tangents contain also Logarithmic Secants, they are to be found as before directed for Sines and Tangents. But because few Tables do, you must, in Case they are wanting, proceed after this Manner; From double the *Log. Radius*, (which is always 20.0000000) subtract the *Log Co-Sine*, the *Log. Secant* will remain. *Example* of 57° 35′.

Thus

Thus from the Double Radius —— 20.0000000
Subtract the Log. Co-Sine of 57° 35′ — 9.7292234
There remains (by *Theo.* XVIII.) the Co-Secant thereof —— } =10.2707766

Precept XI. If you have a Logarithm, and would find the Number; or if it be the Logarithm of a Sine, Tangent, *&c.* and you would find the Degrees and Minutes anſwer thereto; proceed with the Logarithm to the Table, and take either the Number, or the Degrees and Minutes which ſtand againſt the Logarithm next leſs than yours. Thus againſt the Logarithm next leſs to 2.8853912 I find the Number 768; and againſt the Logarithmic Sine next leſs than 9.9652379 I find 67° 22′.

Note. The Places of Figures in the Number ſought, muſt be always one more than the Index of the Logarithm.

Precept XII. In your Operations, if Radius be the firſt Term in your Proportions, you perform then by Addition only; which may alſo be done in any other Caſe; if inſtead of the firſt Term, you put its Arithmetical Complement (which is nothing but its Complement to 10.0000000) then add all the three Terms together, and the Sum (rejecting Radius) is the fourth Term which is ſought. The Arithmetic Complement is moſt eaſily had, by mentally ſubſtracting each Figure of the Log. from Nine, and the laſt from Ten; beginning from the Left-hand, thus the Arithmetic Complement of the Logarithm 4.5882892 is 5.4117108, and 'tis moſt ingenious to work this way, as well as moſt expeditious.

Theſe twelve *Precepts* muſt be well imprinted in the Memory of all who would be ready and dexterous in this moſt excellent Art. Now to the Matter in Hand directly, *viz.* The Reſolution of Triangles according to various Methods above-mentioned.

CHAP

CHAP. VII.

Of the First Method of Solving Right-angled Plain Triangles, by Artificial Sines, Tangents and Secants.

IT will be ſufficient to exemplify and illuſtrate this and ſome other Methods, in the Solution of the firſt Caſe only; ſince by making each Side Radius therein all the Variety of Proportions by Sines, Tangents and Secants, will come into Uſe, as is evident in the Synopſis.

And in order to make the whole Affair plain and evident to the young and untaught Faculties of *Tyro's*, I ſhall proceed in an unuſual Manner, and by a new kind of Scheme, whereby the Reaſon of every Part of the Operation will be obvious, and eaſy to be underſtood

The Scheme is a Quadrant of a Circle, in which is diſcribed a Triangle, conſiſting partly of black and partly of dotted Lines, *viz.* aBc, the very ſmall Part thereof terminated by black Lines, *viz.* ABC, repreſents the Triangle given to be reſolv'd, and is like or ſimilar to firſt Great One, which may be call'd the Original, or Tabular Triangle; becauſe all the dotted Part repreſents that which is contain'd in the Tables. And becauſe every Side of the Original or Tabular Triangle is known in the Tables, and the given One in every Part ſimilar thereto, therefore the *Quæſita* of the propoſed Triangle is found by ſuch Analogy or Proportion; as *per Theor.* XV. and *Precept* V.

The

The Sides of the Original Triangles are computed in ſuch Parts as the Radius conſiſts of 10000000000, the Logarithm of which Number 10.0000000, as in the Table. Accordingly the other Sides, as they are Sines or Tangents, conſiſts of leſs or more of thoſe Parts; and the Judicies of their Logarithms are leſs or greater likewiſe.

But notwithſtanding the Sides of the Original Triangle are calculated to Eight, Nine, Ten, &c. Places of Figures, for which Number of Places alſo the Logarithms are made and fitted; yet becauſe in working Proportions the Truth may be attained to, with a ſufficient Exactneſs, by a leſſer Number of Figures, therefore the Natural Numbers in the Tables exceed not Seven or Eight Places in general.

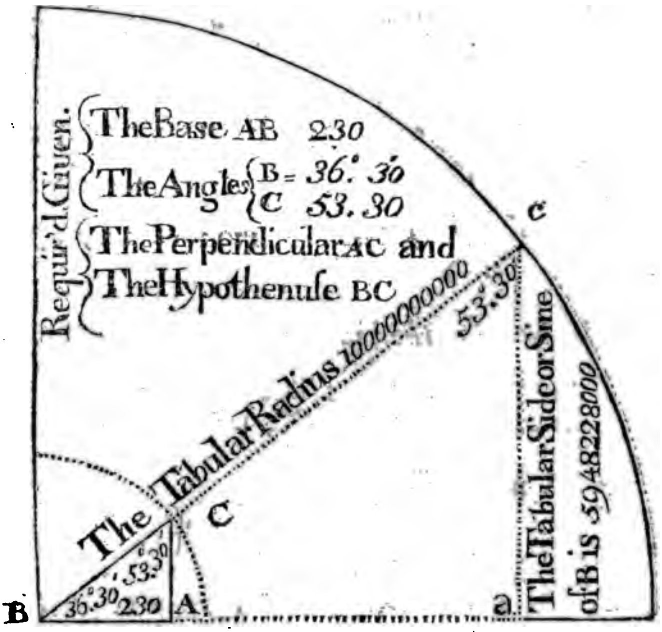
And therefore (for *Example*) when in the Artificial Canon, you ſee the Log. Sine 36° 30′ to be 9.7743876; whoſe Index being 9, ſhews its Natural Number to conſiſt of Ten Places; but yet in the Natural Canon, you find only the Number 5948228 to expreſs that Sine, which Number hath but Seven Places; therefore the three dificient Places muſt be ſupplied by Cyphers, and then the Number will be 5948228000, compleat in Number of Places, though defective in its Figures or Value, yet ſufficient for Uſe.

Theſe Things premiſed diſplay the whole Nature and Miſtery of Trigonometrical Calculation; to which I now proceed.

Caſe

Case I. *The* Hypothenuse *made* Radius.

Triangle I.



The Tabular Side, or Sine of c, 8038569000.

The Analogy for the Perpendicular AC.

As sC : AB : : sB : AC, in the foregoing Syn. That is, As aB : AB : : ac : AC, in the Scheme, by *Theorem* XV.

In Words thus;

		Logarithms
As the Tabular Side, or Sine of c 53′ 30°, in the Original Trian.	*viz.* aB=8038569000=	9.9051787
Is to the Side or Sine of C=c, in the given Tri.	AB = 230 =	2.3617278
So is the Tabular Side, or Sine of B 36′ 30° in the Orig. Triangle	ac = 5948228000 =	9.7743876
		12.1361154
To the Side or Sine of B, in the given Tri	AC = 170,2 =	2.2309367

The Analogy for the Side BC, *or* Hypothenuse.

As sB : AC : : R : BC. That is,
As ac : AC : : Bc : BC, in the Scheme.

In Words,

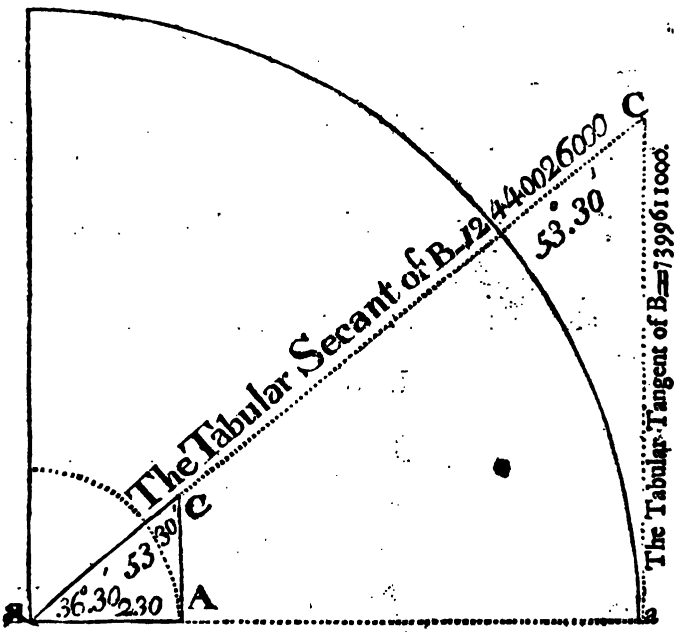
As the Tabular Side or Sine of B 36′ 30° in the Orig. Trian.	ac = 5948228800=	9.7743876
Is to the Side or Sine of B, in the given Triangle.	AC = 170,2=	2.2309367
So is the Tabular Radius ———	Bc= 10000000000 =	10.0000000
To the Hypothen. made Radius, in the given Triang.	BC = 286,1 =	2.4565491

Thus the Measures of the Sides AC and BC, of the given Triangle, are found by Sines, or Proportion of Sides so call'd, in either Triangle; and by the same *Data*, the same Sides are to be found by Tangents and Secants. thus;

CASE

Case I. *The* Base *made* Radius.

Triangle II.



The Tabular Radius 10000000000.

The Analogy for the Perpendicular AC.

As R : AB :: tB : AC. That is,
As aB : AB :: ac : AC, in the Scheme.

In Words,		*Logarithms*
As the Tabular Radius ——	aB=10000000000	=10.0000000
Is to the Base, made Radius —	AB = 230	= 2.3617278
So is the Tabular Tangent of B = 36′ 30°. ——	ac = 7399611000	=9.8692089
To the Tangent Perpendicular —	AC = 170.2	=2.2309367

The Analogy for the Hypothenuse BC.

As R : AB : : seB : BC. That is,
As aB : AB : : Bc : BC, in the Scheme.

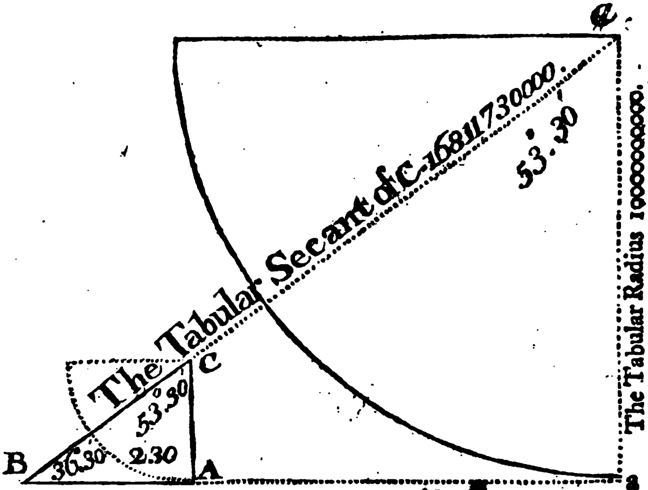
In Words,		
As the Tabular Radius ——	ab=10000000000	=10.0000000
Is to the Base made Radius —	AB = 230	= 2.3617278
So is the Tabular Secant of B = 36′ 30°. ——	BC=12440026000	=10.0948213
To the Secant Hypothenuse —	BC = 286,1	=2.4565491

Thus the Dimensions of the same Sides are found by the Proportions of Tangents and Secants, as was before found by Sines; the same thing will happen when the Perpendicular is made Radius; as in the following Scheme.

CASE

Caſe I. Perpendicular *made* Radius.

Triangle III.



The Tabular Tangent of c=13514224000.

The Analogy for the Perpendicular AC.

As tC : AB :: R : AC. That is,
As aB : AB :: ac : AC, in the Scheme.

In Words,		*Logarithms*
As Tabular Tang. of c = 53′ 30″	aB=13514224000=	10.1307911
Is to the Tangent Baſe	AB= 230=	2.3617278
So is the Tabular Radius	ac=10000000000=	10.0000000
To Radius Perpend. AC =	170,2=	2.2309367

The

The Analogy for the Hypothenuſe BC.

As R : AC :: ſecC : BC. That is,
As ac : AC :: Bc : BC, in the Scheme.

In Words,

As the Tabular Radius ———	ac=	10000000000=	10.0000000
Is to Radius Perpendicular ———	AC =	1792,2=	2.2309367
So is the Tabular Secant of c = 53′ 30°. ———	Bc=	16811730000=	10.2256124
To the Secant Hyp. BC=		286,1=	2.4565491

Thus the Young Trigonometer may ſee the Harmony and Agreement of all the various Ways of working by *Sines*, *Tangents* and *Secants*; the ſame Concluſions reſulting equally from each particular Method.

The *Logarithm Secants* (if not in your Tables) may be found as directed in *Precept* X. But Proportions by Secants are ſeldom neceſſary, eſpecially to the Skillful; and I have only given Example thereof, for Variety and Example's ſake.

I ſaid before, that having illuſtrated the Firſt Caſe of *Plain Right-angled Triangles*, in all the Variety of Methods and Analogies, it would be needleſs to do any more; the other Caſes in the foregoing Synopſis being wrought after a like Manner, and their Proportions being there ſpecified, muſt of Courſe be eaſy to thoſe who underſtand this firſt Caſe; eſpecially as I have now explained and exemplified it by a new and moſt demonſtrative Method.

CHAP.

CHAP. VIII.

Of the Second Method of Solving Right-angled Plain Triangles, by Natural Sines, Tangents and Secants.

THIS Method though first in Nature and Use, I have here placed Second in Order; the Reason why *Logarithmetic Calculation* is allowed Priority of Order has been already assign'd; and the Reason why the Method by *Natural Sines, Tangents,* &c. has Precedence of other more useful Methods which follow, is on Account of its being the Original, most Natural, and the Foundation of most others

This Chapter cannot be so generally Useful as the foregoing, by Reason this Method of *Natural Sines,* &c. is now grown obsolete and unused by any Persons, excepting only such as either have not any Table of *Artificial Sines, Tangents,* or else knows not how to use them; and for their Sakes I shall illustrate this Method by all the Varieties of *Sines, Tangents* and *Secants,* in the Resolution of the First Case of *Right-angled Triangles,* as in the last Chapter.

The Schemes and Analogies there used, will likewise serve here; the Natural Numbers (without the additional Cyphers) as they are taken from the Tables, are here only used.

Case

Case I. Scheme I.

The Hypothenuse *made* Radius.

Given	The Base AB=230 The Ang. B=36′ 30° / C=53′ 30°	Req. the	Perpend. AC and Hypoth. BC

The Analogy for the Perpendicular AC.

As sC : AB : : sB : AC, in the Synopsis.
That is, As aB : AB : : ac : AC, in the Scheme.

The same in Numbers, *with the* Operation.

As 8038569 : 230 : : 5948228 : 170,178, &c.

```
            5948228
                230
           --------
          178446840
          11896456
          ---------
8038569) 1368092440 (170,178,&c.=AC.
         8038569
         --------
         56413554
         56269983
         --------
           1435610,0
            8038569
           ---------
           63175310
           56269983
           --------
            69053270
            64308552
            --------
             4744718
```

The

The Analogy for the Hypothenuſe BC.

As sB : AC :: R : BC, in the Synopſis.
As ac : AC :: Bc : BC, in the Scheme.

The ſame in Numbers; *with the* Operation.

As 5948228 : 170.178 :: 10000000 : 286.08, *&c.*

```
         170.178
         10000000
         ------------
5948228) 1701780000  286.08, &c. = BC.
         11896456
         --------
          51203440
          47585824
          --------
           36176160
           35689368
           --------
             48679200
             47585824
             --------
              1093376
```

The Dimenſions of the Sides of the Triangle, are by this Method found to be the ſame as by the laſt, very nearly; For if great Exactneſs be required, or many Places of Decimals, 'twill be ſureſt and ſafeſt, ſometimes, to work by this Method of Natural Numbers.

The Baſe *made* Radius; Scheme II.

The Analogy for AC.

As R : AB :: tB : AC. That is,
As aB : AB :: ac : AC, in the Scheme.

The same in Numbers; *with the* Operation.

As 10000000 : 230 :: 7399611 : 170.19, *&c.*

```
                      230
               ----------
                221988330
               14799222
               ----------
10000000)      1701910530 (170.19, &c.=AC
```

The Analogy for BC.

As R : AB :: seB : BC. That is,
As aB : AB :: cB : BC, in the Scheme.

The same in Numbers; *with the* Operation.

As 10000000 : 230 :: 12440026 : 286.12, *&c.*

```
                      230
               ----------
                373200780
               24880052
               ----------
10000000)      2861205980 (286.12, &c.=BC
```

The Perpendicular *made* Radius. Scheme III.

The Analogy for AC.

As tC : AB :: R : AC. That is,
As aB : AB :: ac : AC, in the Scheme.

The

The same in Numbers; *with the* Operation.

As 13514224 : 230 : : 10000000 : 170.19, *&c.*

```
                    230
                ----------
13514224)   2300000000 (170.19, &c. = AC.
            13514224
            ----------
             94857760
             94599568
            ----------
              25819200
              13514224
             ----------
              123049760
              121628016
             ----------
               1421744
```

The Analogy for BC.

As R : AC : : seC : BC. That is,
As ac : AC : : Bc : BC, in the Scheme.

The same in Numbers; *with the* Operation.

As 10000000 : 170.191, *&c.* : : 16811730 : 286.12, *&c.*

```
                                  170.191
                                 ---------
                                 16811730
                               15130557
                               1681173
                            117682110
                            1681173
                           ---------------       [=BC.
10000000)       286.1205140430 (286.12, &c.
```

 Thus

Thus is the Firſt Caſe, in all its Varieties, reſolved by the *Method of Natural Numbers*; and after this manner may all the other Caſes be reſolved; there being nothing new or different therein from what is here done.

And, as I have hinted before, this Method may in ſome Caſes be more preferable than the foregoing by *Logarithms*; for though that be moſt expedite and eaſy, yet this gives the Anſwer with greateſt Exactneſs; eſpecially where the Numbers of the given Triangle be large, and the Decimal Parts required to 4 or 5 Places, which ſometimes does happen.

It muſt be obſerved alſo, that when Radius is the Firſt Term in the *Analogy*, the Solution will be moſt exact, and the Operation more eaſy; as may be ſeen in *Scheme* II and III.

CHAP. IX.

Of Solving Right-angled Plain Triangles by the Third Method, viz. *by the Trigonometrical Sliding-Rule.*

THIS *Third Method* by the *Sliding Rule* is the moſt ready and practical Way of any; and is for the moſt part pretty exact; though when the Quantity of the Angle (of which you uſe the Sine or Tangent) exceeds 40 or 50 Degrees; then great Exactneſs muſt not be expected by the common ſized Rule, which is a Foot long. But the longer the Rule is, the better or more exact and uſeful it will be

be of Consequence; and where much Practice in Trigonometry happens, it may be worth while to have one made 4, 5, or 6 Feet long.

A Description of the Sliding Rule.

This Rule, as I have said, is generally about 12 Inches in Length, and does consist of two Parts, *viz.* a fixed and a moveable Part or Slider; on each of which are certain graduated Lines, serving for divers Purposes; As 1. A Line of Inch Measure on one of the Edges. 2. A Line of Equal Parts, marked at the Beginning *EP*. 3. Adjoined to that, a Line of Meridional Parts, for graduating *Mercator's Chart*, marked *M*. 4. A Line of Leagues, marked *Leag.* and, 5. A Line of Longitudes fitted thereto, marked *M. Lon.* 6. A Line of Rhumbs fitted thereto. 7. A Line of Chords, the one marked *R.* the other *C.* The four Lines last mentioned, serve to the Uses of Navigation. 8. On the other Side the Rule, on one Extremity is another Line of Inches; and, 9. On the other Extremity, a Line of the same Length, graduated into 100 equal Parts; by means of these two Lines, is given by Inspection, the Decimal Parts answering to any Number of Inches and Parts of an Inch. 10. On one Side the Rule, on each Side the Grove, is placed a Line of Numbers, or *Gunter's Line*, marked *N.* 11. On the other Side the Rule, on one Side the Grove, is a Line of Sines, marked *S.* 12. On the other Side the Grove, is a Line of Tangents, marked *T.* 13. On one of the Slider or moveable Piece, is a Line of Sines; and, 14. A Line of Tangents, both marked as before. 15. On the other Side the Slider, is a Line of Numbers; and, 16. A Line of Rumb-Sines; and these Lines I have now described are all that are usually put on the *Sliding Rule*, or Instrument here spoken of.

Of

Of all thoſe Lines, the *Lines of Numbers*, *Sines* and *Tangents*, are the only ones uſed in ſolving a *Plain Triangle*; and becauſe it will not a little conduce to form a right Notion of uſing them, I ſhall hint a Word or two concerning their Make and Conſtruction: The *Line of Numbers* is Nothing but the *Logarithms* of the *Natural Numbers*, taken out of the Tables and laid on the Line, not regarding the Indices of the *Logarithms*. Thus for the greater Diviſions of the ſaid Line, *viz.* from 1 to 10; the *Logarithms* to be taken from a Scale of equal Parts, for each Diviſion on the Line, ſtand thus,

Diviſions	2.	3.	4.	5.	6.	7.	8.	9.
Logarithms	.301	.477	.602	.698	.778	.845	.903	.954

For the leſſer Diviſions between 1 & 2, 2 & 3, &c. thus,

Diviſion	1.1.	1.2, &c.	2.1	2.2, &c.	3.1	3.2	&c.
Logarith.	.041	.079, &c.	.322	.342, &c.	.491	.505	

This Conſtruction of the Line of Numbers being very well underſtood, as I preſume it eaſily may; the Conſtruction alſo of the Lines of Sines and Tangents on the Rule is thence ſufficiently evident; for ſince the Artificial Sines and Tangents are but the Logarithms of the Natural Numbers expreſſing the ſame Things, therefore it follows, that thoſe Artifical Sines and Tangents, in the Manner before taught, may be laid down on the Scale, and there form the Lines we now ſpeak of. And which I doubt not but the ingenious Young Trigonometer will eſteem it only his Diverſion to do.

The Deſcription, Nature and Conſtruction of theſe Lines, being thus premiſed, will make the Directions for their Uſe the ſhorter, and Reaſon thereof moſt plain and obvious. From hence alſo it appears that both the preceeding Methods, are in Subſtance, the ſame

ſame with this, in a diverſe Manner apply'd: But whereas in them Exactneſs is the greateſt Thing to be looked to, ſo Eaſe and Expedition are the chief Properties of this *Third Method by the Sliding Rule.*

One Thing I muſt not omit, and that is, to acquaint the young Reader, that as there is no Line of Secants on the Rule, ſo he muſt always obſerve to frame ſuch Proportions as admit of only Sines and Tangents, when he works by the *Sliding Rule*; for as there is ſeldom any Neceſſity for uſing Secants, ſo there is very rarely any Occaſion for them; and when there is, I ſhall ſhew a Means to reſolve it by the Rule nevertheleſs.

In order to perform Operations by the *Sliding Rule*, I ſhall call that Line of Numbers on the Rule it ſelf A, but that Line on the Slider B; alſo I call the Line of Sines on the Rule F, and that on the Slider S. Alſo *Note*, That the End of each Line, *viz.* of Sines and Tangents, at 90° in the one, and at 45° in the other, is Radius in Analogies wrought this way.

Here alſo the foregoing Schemes and Analogies are to be uſed, and needs not either of them to be here again repeated; the manner of operating the firſt Caſe in each Scheme by the Line of Artificial Sines and Tangents now follow.

Caſe I. Scheme I. *To find the* Perpendicular AC.

Direction I. Cauſe the Line of Sines to ſlide by each other, on one Side the Rule; then will the Lines of Numbers do ſo on the other Side. II. Set 53° 30′ on S, to 36° 30′ on F; then, III. Look on the other Side, and againſt 230 on B, is 170.19, &c. on A, the Length of AC, as before.

To

To find the Hypothenuſe BC.

Direct. I. The Rule and Slider being the ſame as before, Set 36° 30′ on S, to Radius (*viz.* 90°) on F; II. Then on the other Side, againſt 170,19, *&c.* on A, is 286.12, *&c.* on B, the Length of BC, as before.

To reſolve this Caſe *by once ſetting the Rule.*

Direct. I. Cauſe the Line of Sines, to ſlide by the Line of Numbers. II. Set 53° 30′ on S, to 230 on A; then againſt 36° 30′ on S, is 170.19, *&c.* on A, for the Perpendicular, and againſt Radius (or 90°) on S, is 286.12, *&c.* on A for the Hypothenuſe. The Reader need not be told that this laſt Way, at once Setting the Rule is much the eaſieſt, beſt, and moſt obvious.

Caſe I. Scheme II. *To find the* Perpendicular AC.

Direction I. Set the Rule ſo that the two Tangent-Lines may ſlide by each other; II. Set Radius or Tangent of 45° on the Slider, to the Tangent of 36° 30′ on the Rule; then, III. On the other Side the Rule, againſt 230 on B, is 170.19, *&c.* on A, for the Length of AC.

Otherways thus,

Direct. I. Make a Tangent-Line and a Line of Numbers ſlide together; then, II. Set Radius (or 45°) to 230 on the Line of Numbers, then againſt the Tangent of 36° 30′ is 170.19, *&c.* as before.

Note, The other Proportion for the *Hypothenuſe* BC, containing a Secant, is to be ſolved by a particular Method of the Rule by and by. Caſe

Case I. Scheme III. *To find* AC.

Direct. I. In this, and all other Cases, when the Quantity of the Angle exceeds 45°, the Learner must observe that on the Tangent Line, the Degrees are number'd from thence back again in smaller Figures; and that these being only the Complements of the other (number'd back in an inverse Order to them,) 'tis manifest that the Slider must be set in the same Place for the Tangent, as for the Co-Tangent; and consequently the Operation the same in all Respects as before, *viz.* in *Scheme* II; only, if Radius be on the Slider, the Answer will be on the Line of Numbers A, & *e contra*. when the two Lines of Tangents slide together,

Note; This Proportion cannot be solved by the Line of Tangents and Numbers sliding together; unless if when you set the Tangent of 53° 30′, to 230 on A, you look as far to the Left of 230 as Radius or 45° is to the Right of it; for there you will see 170.19, &c. For had the Line of Tangents been continued directly forwards, this Proportion might have been solved this Way, as well as the last.

Case IV. Scheme II. *To find the* Angle B.

Direction I. Let Lines of Tangents slide by each other. II. On the Side on which are the Lines of Numbers, set 230 on B, to 170.19 on A; (as the Analogy in the Synopsis directs.) III. And then on the other Side the Rule, against Radius or Tangent of 45°, you find the Tangent of 36° 30′, the Quantity of the Angle B, required.

Note, The second Way by making Tangents and Numbers slide by one another, is only the Reverse of *Case* I. *Scheme* II. and so affords no real Variety;

N the

the ſame may be ſaid of the next Analogy, or *Scheme* III of this preſent *Caſe* IV. Therefore I proceed to inſtance next in

Caſe V. Scheme I. *To find the* Angle C.

Direction I. Make the two Lines of Sines, and the two Lines of Numbers, ſlide together. II. Set 286.12 on A, to 230 on B; then, III. On the other Side, againſt Radius or 90° on F, is the Sine of 53° 30′ on S, the Quantity of the Angle C, as was required.

The Second Way,

Direct. I. Let the Line of Sines ſlide by a Line of Numbers; then, II. Set 286.12 on A, to the Radius or Sine of 90°; then againſt 230 on A, is the Sine of 53° 30′ on S; as before.

Theſe *Caſes* and *Analogies*, which I have hitherto exemplified, contain all the different Ways of uſing the *Sliding Rule*, that at preſent I am apprehenſive of; and am fully aſſured that any one who underſtands what has been here ſaid, cannot avoid being able to uſe the Rule readily on any Occaſion whatſoever.

As thoſe Inſtruments I have now ſhewn the Uſe of, are not ſo very common, as thoſe Rules uſed by Mechanicks, on which are the ſame Lines of Numbers, but not of Sines and Tangents; and as thoſe Perſons do not generally (indeed but very rarely) underſtand the Uſe of Logarithms, or Trigonometrical Calculations thereby; and laſtly, becauſe 'tis eaſy for them to have and to underſtand the Table of Natural Sines, Tangents and Secants; I cannot but think 'twill be acceptable to ſuch, to ſhew how by the common Sliding Rule and a Table of Natural Sines,

Tangents

Tangents and Secants, any Triangle may be solved, with considerable Exactness.

To do this, is no more than to work the Operations by the Sliding Gunter, which in the last Chapter were wrought Arithmetically. In order to this, it must be observed that though the Sine, Tangent or Secant in the Table is expressed by many Figures, yet working by the Sliding Rule, 3 or 4 of the first Figures of those large Numbers are sufficent. For Instance, the Sine of 36° 30′ is 5948228; the Tangent is 7399611; the Secant is 12440026; But 594, 739, and 1244, will suffice for mechanick Purposes wrought by the common Gunter; and by this means Secants may be as well used by the Sliding Rule as Sines and Tangents. I shall illustrate the First Case in all its Variety of Proportions this Way.

Case I. Scheme I. *To find the* Perpendicular AC.

The Analogy for this, in the last Chapter, in Numbers stands thus; 8038569 : 230 : : 5948228 : the Side AC.

But this Analogy, for the Rule, may be thus contracted;

As 803 : 230 : : 594 : the Side AC. Therefore, Set 803 on A, to 230 on B; then against 594 on A, is 170.19, &c. on B, the Length of the Side AC.

To find the Hypothenuse BC.

The Analogy in the tabular Numbers stand thus; As 5948228 : 170.19, &c : : 1000000 : the Side BC. Contracted thus; 594 : 170.19 : : 1000 : the Side BC. Therefore by the Rule; Set 594 on A, to 170.19 on B; then against Radius (or 1000, on the End of A.) is 286.12 on B, the Length of the Side BC.

Case I. Scheme II. *To find the* Perpendicular AC.

The tabular Analogy, is 10000000 : 230 : : 7399611 : to the Side AC.

Contracted for the Rule thus, 1000 : 230 : : 739 : to the said Side.

Therefore, Set Radius on A, to 230 on B, then against 739 on A, is 170.19 on B, the Answer.

To find the Hypothenuse BC.

The tabular Analogy is 10000000 : 230 : : 12440026 : the Side BC.

Contracted for the Rule thus, 1000 : 230 : : 1244 : to the Side BC.

Therefore, Set the first Radius of the Double Line on A, to 230 on B; then against 1244 on A, is 286.12 on B, the Length of the Hypothenuse required.

Case I. Scheme III. *To find the* Perpend. AC.

The tabular Analogy is 13514224 : 230 : : 1000000 : to AC.

That is, Contracted, 1351 : 230 : : 1000 : to the said Side.

Wherefore, Set the first, 1351 on A, to 230 on B; then against 1000 on A, is 170.19 on B, = Length of AC.

To find the Hypothenuse BC.

The tabular Analogy 10000000 : 170.19 : : 16811730 : to BC.

The same Contracted, 1000 : 170.19 : : 1681 : to the said Side.

Now, Set 1000 on A, to 170.19 on B; then against 1681 on A, is 286.12 on B; the Length of the Side BC.

From

From what I have here ſaid concerning this Caſe I. 'twill be very eaſy, in the ſame Manner, to ſolve any of the other following Caſes; which therefore I leave to the young Learner's Exerciſe and Diverſion.

Hence 'tis manifeſt this moſt excellent Art might be much more generally underſtood, and uſed not only by Scholars, but by every common Trader, Artificer or Husband Man, by the cheap and eaſy Means of a Table of *Natural Sines*, *Tangents* and *Secants*, and a common *Sliding Rule*.

And I am ſtrangely ſurpriſed at the ſupine and ſtupid Indolency of many young Perſons, who have (and might have by good Husbandry) Time enough on their Hands, Money enough in their Pockets, and Intellects enough in their Heads, yet notwithſtanding this, will expend neither in the Study of this, or any other Noble Art, or Part of Mathematical Learning; which would in ſo moſt agreeable a Manner, enoble their Nature; enrich their Minds; and elevate them above, and reſcue their Reputations from the rude and barbarous Vulgar. Inſtead of which they idly chuſe to bury their Talents, and wretchedly live, and ignominouſly die without Remembrance.

CHAP. X.

Of the Fourth Method of Solving Right-angled Plain Triangles, by Gunter's *Scale and Compaſſes.*

THIS celebrated Line of *Artificial Numbers*, (whoſe Conſtruction and its Uſe on the *Sliding Rule*, were ſhewn in the laſt Chapter) firſt received its

its Name and Being, from the famous Professor of *Geometry* at *Gresham College*, Mr *Gunter*; of whom it has been commonly called *Gunter's Line*, *Gunter's Scale*, or simply, *The Gunter*: But what I here call *Gunter's Scale*, is a Line of Artificial Numbers, Sines and Tangents, laid down on a Plain Scale or Rule, and are unmoveable.

On this Scale, the same Things are performed by the Compasses, as on the Sliding Scale or Rule, by the Sliding Piece, and the whole Method depends on this easy

General Rule.

Set one Foot of the Compasses in the first Term of the Analogy, (be it Number, Sine or Tangent,) and extend the other Foot (to the Right or Left) 'till it fall on the Term of the Analogy that is with the same Kind with it self (whether it be the Second or Third); and that Extent of the Compasses will reach from the remaining Term) the same Way as before) to the fourth Term or Answer.

The same *Things*, or the same *Schemes* and *Analogies* are to be here used, as before, in solving the first Case.

Case I. Scheme I. *To find the* Perpendicular AC.

Set one Foot of the Compasses in the Sine of 53° 30′, and extend the other to the Sine of 36° 30′; then that Extent will reach from 230 to 170.19, in the Line of Numbers, and is the Answer.

To find the Hypothenuse BC.

Set one Foot in the Sine of 36° 30′, and extend the other to Radius or 90°; then that Extent will reach from 170.19 to 286.12, in the Line of Numbers for Answer.

Case

Cafe I. Scheme II. *To find the* Perpendicular AC.

Set one Foot of the Compaffes in the Tangent Radius or 45°, and extend the other to the Tangent of 36° 30′; that Extent in the Line of Numbers will reach from 230 to 170.19=AC, as before.

To find the Hypothenufe BC.

Note, That becaufe (as I have faid before) there is no Line of Artificial Secants, and this Analogy containing a Secant, it can only be folved by the Line of Numbers with the Compaffes here, as with the fame Line and Slider in the foregoing Chapter, from the Natural Numbers in the Table. Therefore fet one Foot in Radius 1000, and extend the other to 230; then that Extent will reach from 1244 to 286.12= BC, as required.

Cafe I. Scheme III. *To find the* Perpend. AC.

Set one Foot of the Compaffes in the Tangent of 53° 30′, and extend the other to Radius or 45; then in the Line of Numbers that Extent will reach from 230 to 170.19 = AC, as before.

To find the Hypothenufe BC.

Becaufe this Analogy is here alfo by Secants, therefore by the Line of Numbers, from the Natural Numbers of the Table, do as before taught; Set 1000 to 170.19; that Extent will reach from 1681 to 286.12 = BC, the Side fought.

Cafe

Case IV. Scheme II. *To find the* Angle B.

In the Line of Numbers, set one Foot in 230, and extend the other to 170.19; then in the Tangent Line, that Extent will reach from Radius or 45°, to 36° 30′, which is the Tangent of the Angle B.

Note, The Side BC is to be found by the Line of Numbers, *&c.* as directed in the *First Case*, *Scheme* II. III.

Case IV. Scheme III. *To find the* Angle C.

In the Line of Numbers, Set one Foot in 170.19, and extend the other to 230; then will that Extent reach from Radius or Tangent of 45°, to the Tangent of 53° 30′, the Quantity of the Angle C.

The Learner that has regularly come thus far, need not, I suppose, be told that though the Tangent of any Angle, and of that Angle's Complement be in the same Point of the Line, as in this Case, 53° 30′ and 36° 30′ are; yet he may easily know which is the Angle required by the Second Term of the Analogy; for if that be greater than the First, the greatest Tangent is the Angle required; as here 230 being greater than 170.19 makes it certain that 53° 30′ is the Angle sought; and the Contrary.

The Reader is supposed in each of these Operations to have his Eye on the Analogies in the Synopsis, by which they are performed. And these being all the Varieties by the Gunter and Compasses; I proceed to the *Fith Method* by the *Sector.*

CHAP.

CHAP. X.

Of the Fifth Method of Solving Right-angled Plain Triangles by the Sector.

BY this most noble and most useful of all Mathematical Instruments, *The Sector*, not only all the common Operations of the Mathematical Sciences are most easily and commodiously performed; but particularly the whole Business of *Trigonometrical Calculation*, of every Kind, is hereby very easy, expedite and perfect; For all manner of Analogies, whether by Numbers, Sines, Tangents, Secants, are alike resolved here.

The Invention of this wondrous Instrument is founded on the 4th *Prop.* of the 6th Book of *Euclid*, or *Theor.* XV beforegoing. Where 'tis demonstrated, that Parallels to the Base of any Plain Triangle, bare the same Proportion to the Base, as the Parts of the Legs above the Parallel, do to the whole Legs. But more of the Nature and Construction of the *Sector* is to be learned from those who have wrote purposely thereon.

The Lines on the *Sector* are various, but I have to do with no more those which serve more immediately to the Solutions of Triangles, and they are those which follow.

First, A Line of Chords on each Leg of the *Sector*; they proceed from the Center to the End of the Legs, where they terminate in a brass Point, at 60°, the Chord thereof being equal to Radius.

Secondly, A Line of Sines, proceeding from the Center on each Leg, and terminating in a brass Point, on the End, at 90°.

Thirdly, A large Line of Tangents, running from the Center to the End of each Leg, and there end in a brass Point at 45°, the Tangent thereof being equal to Radius.

Fourthly, A small Line of Tangents, beginning at a small Radial Distance from the Center on each Leg, and terminates at the End at about 75° or 76°; and is not perfect, as being only a Supplement to the former.

Fifthly, A Line of Secants, beginning at some distance from the Center, and proceed to the End on each Leg, where they break of at, some 60°, some 75°, *&c.*

Sixthly, A Line of Equal Parts, or, as some term it, a Line of Lines; these proceed from the Center on each Leg, to the End; graduated by equal Divisions 1.2.3.4, *&c.* to 10.

The Line of Chords, on the End of each Leg, is marked with a C. the Line of Sines, with S. the Line of Great Tangents, with T. the Line of Small ones, with *tan.* the Line of Seconds, with *sec.* and the Line of Equal Parts, with I.

These are the Six Grand Lines, which are of most important Use; the Construction of each of which you have clearly and largely taught by the foregoing XIX *Problem of Geometry.*

These are all Lines of Natural Numbers, Sines, Tangents and Secants; having now pretty well done with the Artificial ones, on which we have dwelt so long.

These Lines are to be used in a twofold Manner, *viz.* Doubly or Singly; Doubly, when we use both Lines of a like Sort; as both the Lines of Lines, both the

the Lines of Sines, &c. as suppose you place one Foot in the Sine of 40° on one Line, and extend the other Foot (the *Sector* being open'd) to 40° on the other Line, on the other Leg of the *Sector*; and this is call'd taking of 40° Parallel-wise, and is, for Brevity's sake, thus marked, =40°.

The second Manner of using these Lines, *viz* singly, is when we use only one Line of a Sort; and Length-ways, or Lateral-wise, take off any Number of Parts or Degrees, by setting one Foot of the Compasses in the Center, and extending the other along one Leg to the designed Point, as suppose 40° on a Line of Sines, and this Lateral Way of taking 40° is thus marked, ‖40°.

When ever, in working Proportions, Radius is mention'd, or made use of, you must understand it to relate to, or be in the Ends of the Great Lines on each Leg, on the brass Pin, as at 60 and 60, for Chords; 90 and 90 for Sines; 45 and 45 for Tangents, and 10 and 10 for the Line of Equal Parts; but on the Small Line of Tangents, the Radius or Radial Point, is the beginning at 45°; as it is also of the Small Secants, at 00°, or very Beginning; but the following Practice will make all things intelligible and clear.

I shall resolve the *First Case* in all its Varieties, as I have all along done, and here again repeat the Analogies with the proper Signs prefixed to each Term, in order to shew how they are to be wrought on the *Sector* by the Compasses.

And here, that the young Learner may meet with nothing unapprised of, he must know, 'tis most convenient to begin with that Lateral Term, or more properly, to make that Term a Lateral one, that when apply'd Parallel-wise, may occasion the least Opening of the *Sector*; and all the following Analogies are marked with Regard hereto.

Case I. Scheme I. *To find the* Perpendicular AC.

The Analogy; As = sC : || AB : : = sB : || AC.

Therefore with your Compasses take the Lateral Distance 230 from the Line of Lines, and make it a Parallel Distance from 53° 30′ to 53° 30′ on the Lines of Sines; Then the Parallel Distance of 36° 30′ and 36° 30′ of Sines will reach Laterally from the Center to 170.19 on the Line of Lines, and gives the Answer.

To find the Hypothenuse BC.

The Analogy; As = sB : || AC : : = R : || BC,

That is, Make the Lateral Distance 170.19 on the Line of Equal Parts, a Parallel on the Sine of 36° 30′; then shall the Radius, or Sine of 90°, extend Laterally from the Center to 286.12 on the Line of Equal Parts, which is the Length of BC.

Case I. Scheme II. *To find the* Perpend. AC.

The Analogy; As = R : || AB : : = tB : || AC.

Make the Lateral 230, a Parallel on the Tangent Radius, or 45°, then the Parallel Tangent 36° 30′, will make Laterally 170.19 on the Line of Equal Parts.

To find the Hypothenuse BC.

The Analogy; As = R : || AB : : = seB : || BC.

Make the Lateral Distance 230 a Parallel on Radius of Secants; then the Parallel Secant of 36° 30′, shall be the Lateral Distance 286.12 on the Line of Equal Parts, as required.

Case I. Scheme III. *To find the* Perpend. AC.

The Analogy; As $=$tC : $\|$ AB : : $=$ R : $\|$ AC.

Make the Lateral Distance 230, a Parallel on the small Tangents, of 53° 30′; then shall the Parallel Radius of the small Tangents, or 45°, be the Lateral Distance of 170.19 on the Line of Equal Parts, the Length of AC.

To find the Hypothenuse BC.

The Analogy; As $=$ R : $\|$ AC : $=$ seC : $\|$ BC.

Make the Lateral Distance 170.19, a Parallel on the Secant-Radius; then the Parallel Secant of 53° 30′, shall be the Lateral Distance of 286.12 on the Line of Equal Parts.

If the Method of thus solving the *First Case* by the *Sector*, be well understood, there cannot possibly happen any difficuly in the Solution of any of the rest after the same Manner; especially as I have here adjoyn'd all the Analogies with their directory Signs.

Case II. Scheme I.

Analo- { 1. As$=$sB : $\|$ AC: :$=$sC : $\|$ AB, the Base.
gies { 2. As$=$sB : $\|$ AC: :$=$ R : $\|$ BC, the Hypot.

Scheme II.

Analo- { 1. As$\neq$tB: $\|$ AC: :$\neq$R : $\|$ AB, the Base.
gies { 2. As$=$ R : $\|$ AB: :$=$seB : $\|$ BC, the Hypot.

Scheme

Scheme III.

Analo- { 1. As=R : || AC: := tC : || AB, the Base.

gies { 2. As=R : || AC: :=seC: || BC, the Hypot.

Case III. Scheme I.

Analo- { 1. As=R : || BC: :=sC : || AB, the Base.

gies { 2. As=R : || BC: :=sB : || AC, the Perpend.

Scheme II.

Analo- { 1. As=seB : || BC: :=R : || AB, the Base.

gies { 2. As=R : || AB: :=tB: || AC, the Perpend.

Scheme III.

Analo- { 1. As=seC : || BC: : || tC : || AB, the Base.

gies { 2. As=tC : || AB: : || R : || AC, the Perpen.

Case IV. Scheme II.

Analo- { 1. As || AB: =R: : || AC: =tB, Ang. at Base

gies { 2. As=R : || AB: :=seB : || BC, the Hyp.

Scheme III.

Analo- { 1. As || AC:=R : : || AB :=tC, Ang. at Per.

gies { 2. As =R : || AC: :=seC: || BC, the Hyp.

Case V. Scheme I.

Analo- { 1. As || BC:=R : : || AB := sC, Ang. at Per.

gies { 2. As =R : || BC: := sB : || AC, the Perpen.

Case

Scheme II.

Analo-gies { 1. As ‖ AB := R : : ‖ BC : = seB, Ang. at Base
2. As = R : ‖ AB : := tB : ‖ AC, the Perpen.

Case VI. Scheme I.

Analo-gies { 1. As ‖ BC : = R : : ‖ AC : − sB, Ang. at Base
2. As = R : ‖ BC : := sC : ‖ AB, the Base.

Scheme III.

Analo-gies { 1. As ‖ AC : = R : : ‖ BC : = seC, Ang. at Per.
2. As = R : ‖ A : := tC : ‖ AB, the Base.

Thus I hope I have made the whole Affair of Trigonometrical Calculation facile and ready by the *Sector*, however the Proportions are Constituted of Numbers and Sines, Numbers and Tangents, or Numbers and Secants; so that though a Person come ever so unskill'd in the Matter, I think when he has duely read and digested the Doctrine of this Chapter, 'tis sufficient to make him perfect, and dexterous in applying this Noble Instrument to all the Purposes here treated of.

Note, I would advise the young Student, when he buyes a *Sector*, to have one 2 Feet in Length when opened strait; for then the Lines will be 12 Inches each, and the Divisions thereof will be larger and clearer; however one of 18 Inches may do pretty well for common Uses. Observe also, that the Lines of Secants and small Tangents begin at an equal Distance from the Center; for else such Analogies as have both Tangents and Secants in them cannot be performed thereby.

CHAP

CHAP. XII.

Of the Sixth Method of Solving Right-angled Plain Triangles by Geometrical Construction.

BY Geometrical Construction is meant a Delineation of the Triangle in Lines by a Scale of Equal Parts and a Line of Chords, by the strict Method of Geometry taught in the *Chapter of Geometrical Problems*; so that when the whole Triangle is thus projected in *Plano*, the Parts thereof unknown may be easily measured by the Scale from which the known or given Parts were delineated; that is, the unknown Sides measured on the Line of Equal Parts, and the Quantity of the unknown Angles from the Line of Chords; and thus the whole Triangle becomes known as soon as constructed.

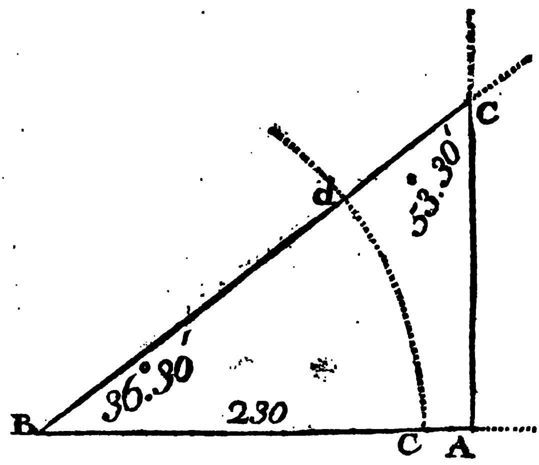
The Scale for this Purpose ought to have a Line of Equal Parts divided Diagonally, such as is common on the *Plain Scale*; however a Line of Equal Parts of some kind or other there must of Necessity be; and also a Line of Chords, or, in lieu thereof, an exact graduated Limb of a Quadrant. Whence this Geometrical Construction or Protraction of a Triangle may be performed by divers Instruments, *viz.* all such as have on them the Lines aforesaid, as the common *Plain Scale*; *The Sector*; *The Protractor*; *Sutton's Quadrant*; &c. But of all these the most usual and useful are, the *Plain Scale* and *Sector*; the

first

first having a Line diagonally divided, which gives an Answer much nearer the Truth, than one that is not thus divided can do; and the Latter, as it is capable of being set to any Radius, which is a Property peculiar to its self.

The Geometrical Construction of the *Six Cases* of *Right-angled Plain Triangles*, according to what is given in each, here follows.

Case I. *Given the* Base *and* Angles.



First, Draw the Base Line BA at pleasure, and from the Diagonal Line on the Plain Scale, or Line of Equal Parts on the *Sector*, take the Length of the Base 230, with your Compasses, and set it from B to A.

Secondly, On A raise a Perpendicular (by *Problem* II.) indefinitely continued.

Thirdly, Take the Chord of 60 Degrees from the Line of Chords on the *Plain Scale*, off from the Limb of the *Protractor* or *Quadrant*; or lastly, from the Parallel Chords of 60° on the *Sector* (conveniently opened) and therewith (as a Radius by *Theorem* XVII.) setting one Foot of the Compasses in B, strike the Arch dc; and from the same Line of Chords take the Quantity of Angle B, *viz.* 36° 30′, and set from c to d.

Fourthly, From B draw a Line through the Point d 'till it meet the Perpendicular in C. Then is ABC the Triangle required.

Fifthly, Measure the Perpendicular AC on the same Line or Scale you took AB from, and it will be found to be 170.19.

Sixthly, Measure also on the same Line the Hypothenuse BC, and you will find it to be 286.12; and thus the whole Triangle is solved.

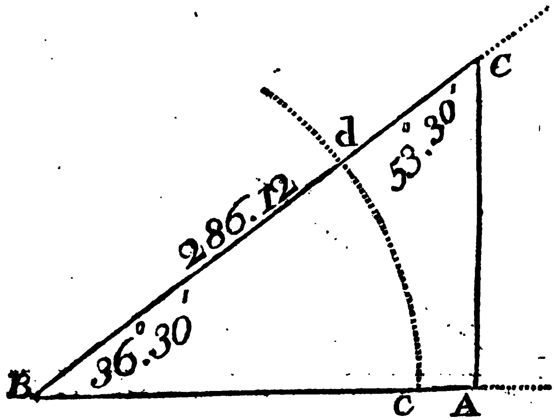
Case II. *Given the* Perpendicular *and* Angles.

This *Case* is constructed in the same Manner as the last, if you proceed with the Perpendicular AC, and the Angle C, as I before directed for the Base AB and Angle B.

Scheme omitted.

Case

Case III. *Given the* Hypothenuse *and* Angles.



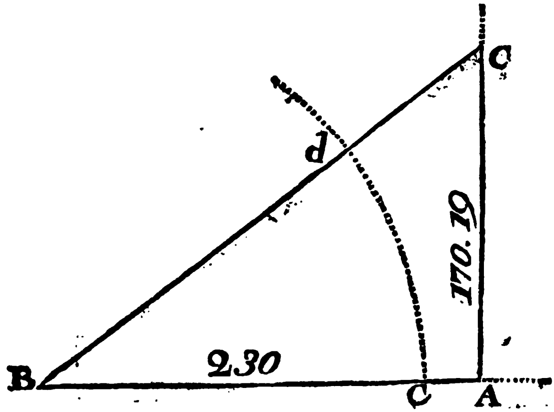
First, Draw a blank Base Line BA with one Foot of the Compasses, on which at B make an Angle, as directed in *Case* I.

Secondly, Through the Point *d* draw the Line BC indefinitely, and from a Scale of Equal Parts take 286.12 and set from B to C thereon.

Thirdly, From the Point C let fall a Perpendicular (by *Problem* III.) to the blank Line BA, cutting it at Right Angles in the Point A, and join A, B for the Base.

Fourthly, Measure the Base BA and the Perpendicular AC, on the Scale, and you will find the first to be 230; and the other 170.19, as before.

Case IV, *Given the* Base *and* Perpendicular.



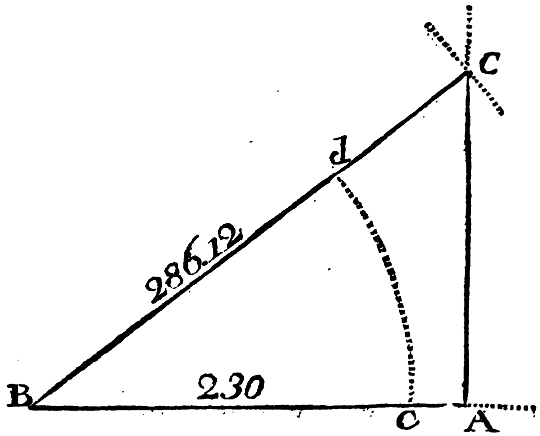
First, Draw the Line BA, on which from a Diagonal Scale or Line of Equal Parts, set off the Base 230 from B to A;

Secondly, On the Point A erect the Line AC at Right Angles, on which set off the Perpendicular 170.19 from A to C.

Thirdly, Joyn the Points B and C, for the Hypothenuse; which measured on the aforesaid Scale, will be found to be 286.13.

Fourthly, With 60 Degrees of Chords on B, strike the Arch c d, and measure the Part c d, on the Scale of Chords, or on the Limb of the Protractor, or apply it Parallel-wise on the Lines of Chords on the Sector, and it will give the Angle B 36° 30′, whose Complement C, is 53° 30′; and thus the whole Triangle is become known.

Case V, *Given the* Base *and* Hypothenuse,



First, Draw the Line BA, on which from a Scale of Equal Parts set off the Base 230 from B to A.

Secondly, On A raise the Perpendicular AC, continued indefinitely.

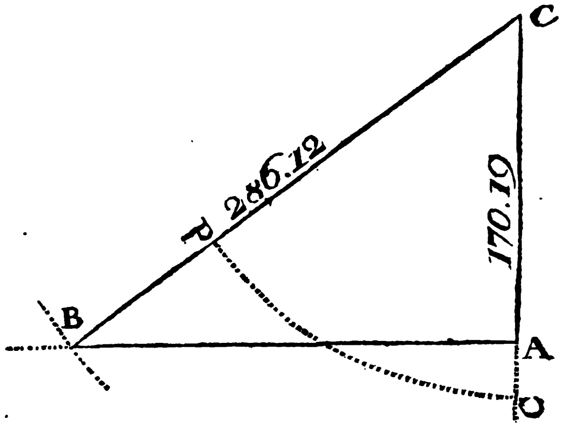
Thirdly, Then from the same Scale of Equal Parts take the Hypothenuse 286.12, and set one Foot of the Compasses on B, with the other cross the Line AC in C; and draw the Line BC.

Fourthly, With the Compasses take AC, and measure it on the Line of Equal Parts, and you will find it 170.19.

Fifthly, With the Chord of 60 Degrees, on B, strike the Arch cd, which being measured on the same Scale of Chords, it will give the Angle B=36° 30′; whose Complement C is 53° 30′.

Case

Case VI. *Given* Hypothenuse *and* Perpendicular.

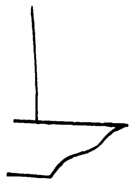


This Case is exactly the same as the foregoing, as to the Method of Construction; if the same Directions be applied here to the Perpendicular and Hypothenuse, as was there given for the Base and Hypothenuse; as is evident from the *Scheme* it self.

Thus I have in a plain and familiar Way, shewn the Geometrical Construction of all the *Cases of Plain Right-angled Triangles*, in order to their Solution; and which ought to be perfectly understood, because 'tis the very Basis of all practical *Trigonometry*; and without which, no such thing as a Triangle can be truly made.

To learn this true and exact Geometrical Way, being so very plain, and easiy in Practice; I have often admired that the ingenious Handicraft and Mechanick should so much neglect it, and be so unreasonably contented with the old unartful Way of drawing Figures by *Tentando*.

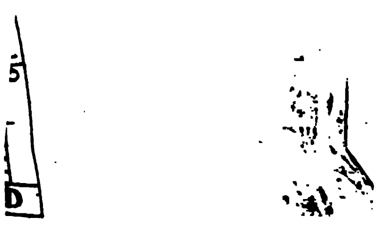
CHAP.



ure. n°

Sinical

drant



CHAP. XIII.

Of the Seventh and Eighth Methods of Solving Right-angled Plain Triangles by the Practical Trigon, and Sinical Quadrant.

THE two Instruments, whose Uses are here to be described, are the most expeditious in Practice of any by Natural Lines; yea, they are in themselves the most natural of any Means; for either at once constitute the Triangle, and shews the Dimensions of every Part thereof, by Inspection only, in Natural Numbers on the respective Lines But because those Instruments are very scarce, and in few Persons Hands; I have therefore given the Young Geometer a Figure both of the *Trigon* and *Senical Quadrant*, and their Descriptions which here follow.

A Description of the Trigon. Fig. I.

The *Trigon* is an Instrument consisting of three, and sometimes of four, Parts or Pieces: The first is the fixed Piece BA, which represents the Base of a Right-angled Triangle, and is graduated into an hundred Equal Parts. The second Piece is BC, and is inserted into BA by the Joint B, on which it moves, and may be set to a given Angle with BA; this Part represents the Hypothenuse of a Triangle. The third

third Piece is AC, and is made to move backward and forward on BA; by means of the Socket S, so as to be in a perpendicular Position thereto; and therefore this Part represents the Perpendicular or Cathetus of the Triangle, and is also graduated into an hundred Equal Parts. The fourth Part is the Semiquadrantal Arch D, divided into 45 Degrees; being fixed in the End of the Part BA, in order that the Piece BC may move commodiously by it, and be set to the given Quantity of any Angle under 45° thereon. By this Description, and even from the very view of the Figure it self, 'tis easy to conceive how very naturally this Instrument at once both forms and shews the Quantity of each Part of the Triangle unknown. The Reason why the Arch is here described as containing only 45°, and no more, will appear in the Uses hereof by and by. And here 'tis to be observed, that because this Instrument may be opened to an Angle of 45°, and each Leg in such Case may contain 100 Parts, 'tis therefore necessary the Hypothenusal Part BC, should be divided into 141.4 of such Parts as being the Square Root of 20000, the Sum of the Squares of the Legs. See *Theorem* XI.

Note, If the Leg AC were made to move on the Socket S circularly, this Instrument might also be used in solving *Oblique-angled Triangles.*

A Description of the Sinical Quadrant.

This Quadrant hath, like all others, a graduated Limb of 90 Degrees; and its two Rectilineal Sides (or Radius's) BD and BE, divided into 100 Equal Parts, from each of which are drawn Right Lines to the Circular Limb, mutually intersecting each other on the Superficies of the Quadrant, and make the Sines and Co-Sines of as many Divisions in the Quadrantal

Quadrantal Limb. On the Center of the Quadrant B is fixed a moveable Index or Label BC, graduated alſo into 100 Equal Parts; this Index thus moving on the Quadrant is to be ſet to any Given Angle, and ſerves for the Hypothenuſe in any Triangle, the other two Legs which make the Baſe and Perpendicular being thoſe Lines, or Parts thereof which ariſe from either graduated Side and meet the graduated Edge of the Label, and the Side of the Quadrant it ſelf; all this is evident by a View of the *Figure* II, only.

It is not my Purpoſe (here at leaſt) to ſhew the other Uſes that may be made of the Quadrant here deſcribed, and the *Trigon*, but only that of ſolving Triangles thereby; and that in the *Trigon* is as follows.

The Uſe of the Trigon.

Admit there be a Right-angled Triangle, in which there is Given the Baſe BA = 82, or 820; and the Angle at Baſe B = 30° 00′; and its Complement of Courſe, C = 60° 00′ required the Hypothenuſe BC, and Perpendicular AC.

Direction I. Set the Hypothenuſal Part BC to 30° 00′ on the Limb, then ſlide the Perpendicular Part AC to 82 (or 820) on the Baſe Part BA; this being done, the Triangle is formed; and from the Center B to the Common Interſection at C, is contained 94.68 (or 946.8) = BC, the Hypothenuſe; and from A to C, is intercepted 47.34 (or 473.4) = AC, the Perpendicular; thus with Eaſe, and in a Moment is ſuch a Triangle ſolved by Inſpection only.

Direction II. If the Angle at Baſe, in the aforeſaid Caſe, be Given greater than 45°; the moveable Hy-

Q pothenuſe

pothenufe muft be fet to the Complemental Angle, and the Inftrument being inverted, *viz.* the Perpendicular Piece made Bafe, the fmall Divifions muft be ufed, as before the larger were, and the Anfwer will appear on the other Parts as before; but not exact enough for any confiderable Purpofe, unlefs the two Legs are at leaft a Foot in Length each; and fuch as chufe this Inftrument, may as well have it 2 or 3 Feet long, as one; and their Work will be proportionably more exact.

Direction III. When both the Legs are Given, do thus; Set the Perpendicular Leg AC to what is Given on the Bafe Leg BA, then move the Hypothenufal Part BC to the Given Parts on AC; and thus is the Triangle formed, and the Solution evident by Infpection; for the Point of the Hypothenufe on the Limb D fhews the Quantity of the Angle at Bafe *B*, and fo of its Complement C; and the Parts intercepted between B and C on the Hypothenufe, are its required Length.

Direction IV. When Bafe and Hypothenufe are Given; flide the Perpendicular Part AC to the Given Part of the Bafe BA, then move the Hypothenufal Part up or down, 'till the Given Part thereon meet the Perpendicular AC; and thus the Triangle is formed, and the Angles and other Side, are known by Infpection.

Direction V. When the Hypothenufe and Perpendicular are Civen; move the Parts BC and AC, fo together, that the Given Parts on both may coincide in C; when this is done the Triangle is formed, and folved by Infpection.

Direction

Direction VI. When the Hypothenuſe and Angles are Given; Set the Hypothenuſal Part BC to the Quantity of the Angle B on the graduated Limb D, then move the Perpendicular Part AC to the Given Part on BC; and thus the Triangle is formed, ſhewing the Quantity of the other two Sides BA and AC.

Direction VII. When the Perpendicular and Angles are Given; Set the Hypothenuſe BC to the Degrees of B on the Limb D, then ſlide the Perpendicular backwards and forwards, 'till its Given Parts meet the Hypothenuſe in C; ſo is the Triangle formed, and the Sides viſible on their reſpective Parts BC and BA.

Thus have I given *Directions* for the Uſe of this Practical Inſtrument in all Variety of Caſes. I proceed now to

The Uſe of the Sinical Quadrant.

In the following *Directions*, I call thoſe Lines which are drawn from the Diviſions of the Side of the Quadrant BD upwards, *Right Sines*; and thoſe which are drawn from the Side BE, acroſs the Quadrant, *Tranſverſe Sines*.

I. *Let there be Given the* Baſe BA=63, *and the* Angle B=38° 30′; *its Complement being* 51° 30′; *to find the Sides* BC *and* AC, *by the* Sinical Quadrant.

Direction I. Set the Index to the Angle B on the Limb; then obſerve where the Right Sine of 63 Parts on BD interſects the Index, and you'll find it to be in C in the Diviſion of 80.5, which therefore is the Length of the Hypothenuſe BC on the Index. Next for the Perpendicular AC, obſerve where the

Point C is transferred to the Side BE, by the Transverse Sine CG, which you will find to be in the Point C in the Division 47.3, the Length of the Part BG= AC, the Perpendicular. See *Theorem* IX.

Direction II. In case a Side or Angle be Given so large, as that the Side exceeds the Larger Divisions; or the Point of Intersection C be carried off the Quadrant; you must use the small Divisions, and proceed altogether as in *Direction* II. for the Use of the *Trigon.*

II. *Given the two* Legs BA *and* AC; *to find the Rest.*

Direction III. Observe where the Right Sine of the Given Point on the Base Side BD intersects the Transverse Sine of the Given Point in the Perpendicular Side BE; then on that Point of Intersection, as C, lay the Index; and so shall the Triangle be formed, and the Quantity of every Part at once appear.

III. *Given the* Base *and* Hypothenuse; *to find the Rest.*

Direction IV. Lay the Given Point in the Index, on that Right Sine which proceeds from the Given Point in the Base Side BD, and the Triangle is formed; in which the Angles will be apparent, and the Perpendicular known by the Transverse Sine, going from the Point C to the Perpendicular Side BE.

IV. *Given the* Hypothenuse *and* Perpendicular; *to find the Rest.*

Direction V. Lay the Index BC in the Given Point thereof C, on the Transverse Side which proceeds from

from the Given Point in the Perpendicular Side BE; ſo is the Triangle conſtituted; and the unknown Parts immediately becomes known.

V. *The* Hypothenuſe *and* Angles *given*; *to find the Reſt.*

Direction VI. Set the Index to the Angle B on the Limb; then obſerve what Right and Tranſverſe Sines meet in the Given Point in the Index; for they ſhall ſhew the Baſe in the Side BD; and the Perpendicular in the Side BE.

VI. *The* Perpendicular *and* Angles *given*; *to find the Reſt.*

Direction VII. Set the Index to the Angle B on the Limb; then ſhall the Tranſverſe Sine cut the Index in the Point for the Hypothenuſe, and the Right Sine deſcending from that Point, ſhall ſhew the Baſe in the Side BD.

Thus by eaſy *Directions*, too plain and natural to need Numerical Examples for each, I have ſhewn how all the *Caſes of Right-angled Plain Triangles* are to be reſolved by the two Inſtruments above deſcribed; which were they ſufficiently large and exact, would be moſt Uſeful in Practical Trigonometry; the firſt of which any ingenious Artiſt might make (at leaſt might graduate) himſelf, of what Size he pleaſe.

CHAP.

CHAP. XIV.

Of the Ninth Method of Solving Right-angled Plain Triangles by Natural Arithmetic.

THE Foundation of this *Method* is from the most famous *Pythagorean* Invention, *viz. Theorem* XI. which teacheth the Arithmetic of Squares, Triangles, Parallelograms, Circles and other similar Superficies described on the Sides of a Right-angled Triangle, by Extraction of Roots.

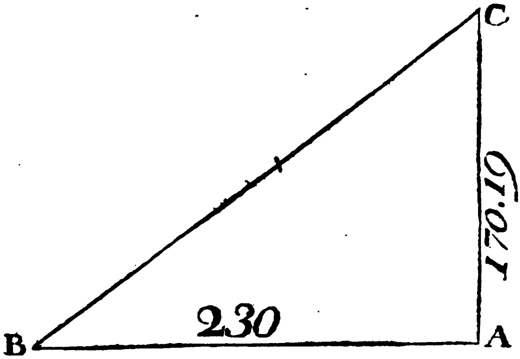
This *Method* (here intended by Extraction of the Square Root) is defective, inasmuch as by it the Angles cannot be found. Yet is it a very useful Means for finding the Sides on many Occasions. And he who knows not, by having two Sides Given, to find the other readily with the Pen only, is but a poor Geometer.

In *Theorem* XIX. 'twas shewn and demonstrated, that by having any two Sides of a Right-angled Plain Triangle Given, the other by Extraction of the Square Root was Given also. Therefore by this *Method* there are only Three *Cases* of the Six, to be solved, *viz.* those Three in which are two Sides Given; and those, as I said, imperfect; because by them no Angle can be known.

The *Solutions* of those Three *Cases* by Extraction of the Square Root are as follows.

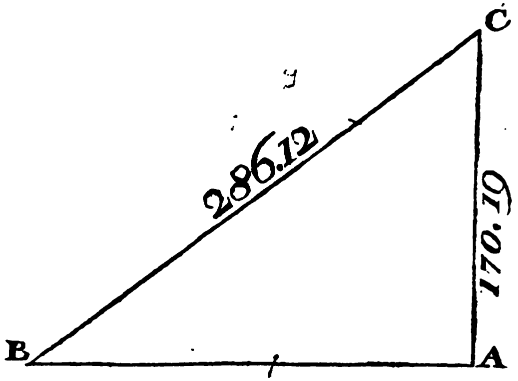
Case

Case I. *Given the* Two Legs BA=230, *and* AC =170.19. *To find the* Hypothenuse BC.




```
     AC=230          AC=170.19
        230             170.19
     ------          ---------
       6900             153171
      460               17019
     ------           1191330
Square of BA = 52900   17019
                     ----------  Squ.
               Add { 28964.6361 = of AC
                   { 52900
                     ----------
Sum of the Squares of BA&AC=81864.6361(286,12
                          4
                         ---
The Square Root of    48) 418
 which is 286.12 fere,    384
 and is the Hypo-    566) 3464
 thenuse sought; as       3396
 before.            5721)  6863
                           5721
                   57222) 114261
                          117444        Case
```

Case II. *Given the* Hypothenuse BC = 286.12, *and the* Leg CA = 170.19. *To find the* Leg BA.



$\left.\begin{array}{l} BC = 286.12 \\ AC = 170.19 \end{array}\right\}$ Squared is $\left\{\begin{array}{l} 81864.6361 \\ 28964.6361 \end{array}\right.$

The Diff. of those Squares = 52900 = BCq — ACq.

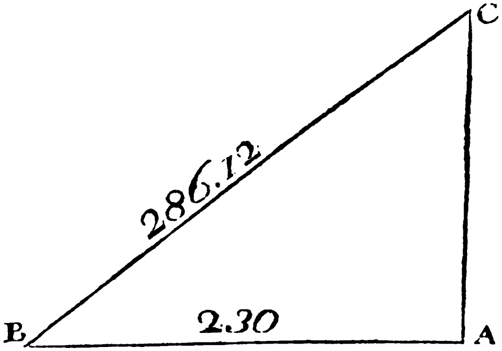
By Extraction. 52900 (230 = BA, the Leg sought.

```
     52900 (230
     4
43)  129
     129
     ...00
```

Note, I chuse to keep to the Numbers before used, though the last Place of Decimals in the Root and Square of BC be not precisely just.

Case

Case III. *Given the* Hypothenuse BC=286.12, *and the* Leg BA=230. *To find the other* Leg AC.



BC=286.12 } Squared is { 81864.6361
BA=230 } { 52900

The Difference of those Squares is 28964.6361

Then, by Extraction } 28964.6361 (170.19=AC, { the Leg sought.

```
         28964.6361 (170.19
         1
     27) 189
         189
  3401)  ...64.63
            34 01
  34029)   306261
           306261
           ......
```

I know there is a kind of *Method* for finding the Angles by *Plain Arithmetic*; but because 'tis very troublesome, intricate and consequently useless, (besides another Reason which shall go nameless) I do not think 'tis worth while to insert it here.

R CHAP.

CHAP. XV.

Of the Tenth Method of Solving Right-angled Plain Triangles by Algebra, or Analytical Investigation.

THIS *Method* of resolving *Triangles* is only to be managed by those who have some Skill in *Algebra*. For those *Cases* which require to be solved this Way, fall not under the common Rules, and consequently will not admit any Solutions in the common *Methods* hitherto treated of. In them the *Data* and *Quæsita*, or the Parts given and required, are intire; that is Whole Sides, or Whole Angles, and that separately; But here you have the *Data* most times consisting of Pieces of Triangles, that is, Parts of Sides, Sums, Differences, and Proportions of Sides and Angles only; and for the Triangle to fall out thus is no very rare Thing with Those who are conversant in *Geometry*, the more abstruser Part especially.

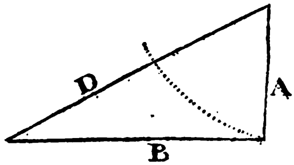
Hence therefore the Young Geometer cannot but think himself concerned to learn so necessary a Part of his Art as the *Analytical Investigation* of the unknown Parts of *Triangles*, and other Figures depending thereon; and though I do not here pretend to teach the Art of *Algebra*, yet I imagine 'twill not be a little acceptable to the *Young Philomath*, to have here a kind of *Synopsis* of all the most usual and useful *Cases* of this Nature laid before him, with the *Theo-*

rems

rems for their Solutions, and a Specimen of the Investigation and Resolution of such *Theorems*; all which here follow in Order.

Triangle

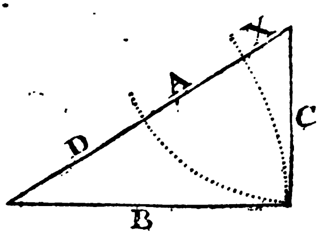
Case I. *Given the* Base B, *and the Difference between the* Hypothenuse *and* Cathetus D; *to find the* Cathetus A, &c.



Theorem

$$A=\frac{BB-DD}{2D}$$

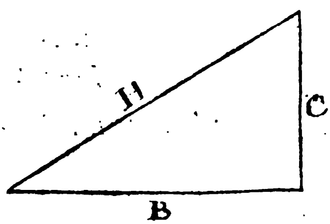
Case II. *Given the Difference of the* Base *and* Hypothenuse D, *and the Difference of the* Cathetus *and* Hypothenuse X; *to find the* Base B, *the* Cathetus C, *and the Part of the* Hypothenuse A.



Theorem, $A=\sqrt{2DX}$

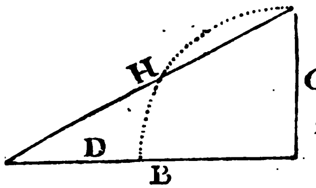
Then, $\begin{cases} D+A=B \\ X+A=C \end{cases}$

Case III. *Given the* Hypothenuse H, *and the Sum of the* Legs (viz. B+C=) S; *thence to find those two* Legs.



1 Theorem. $B=\dfrac{S+\sqrt{2HH-SS}}{2}$

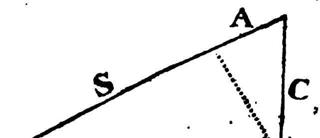
2 Theorem. $C=\dfrac{S-\sqrt{2HH-SS}}{2}$



Cafe IV. *Given the* Hypothenufe H, *and the Difference of the other two* Sides D; *to find the* Sides B *and* C.

1 Theorem. $B=\dfrac{D+\sqrt{2HH-DD}}{2}$

2 Theorem. $C=\dfrac{\sqrt{2HH-DD}-D}{2}$



Cafe V. *Given the* Bafe B, *and the alternate Segment of the* Hypothenufe (*made by a* Perpendicular *falling from the* Right-angle *thereon*) S; *to find the other* Segment A, *and the* Cathetus C.

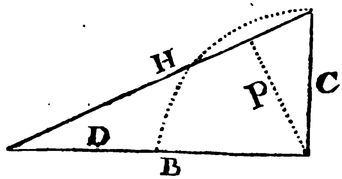
1 Theorem. $A=\sqrt{BB-\frac{1}{4}SS}-\frac{1}{2}S.$

Let $S+A=H$; then

2 Theorem. $C=\sqrt{HH-BB}.$

Cafe

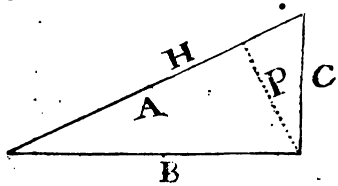
Case VI. *Given the Difference of the* Base *and* Cathetus D, *and the* Perpendicular *let fall from the* Right-Angle *on the* Hypothenuse P; *thence to find the* Hypothenuse H, &c.



Theorem. $H=P+\sqrt{DD+PP}$.

And now B and C are found by *Case* IV.

Case VII. *Given the* Hypothenuse H, *and the* Perpendicular P, *let fall from the* Right-Angle; *thence to find the Greater* Segment *of the* Hypothenuse A, &c.

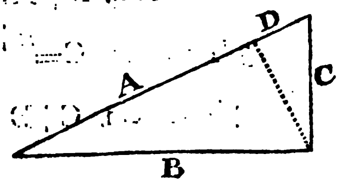


1 Theorem. $A=\frac{1}{2}H+\sqrt{\frac{1}{4}HH-PP}$.

2 Theorem. $B=\sqrt{AA+PP}$.

3 Theorem. $C=\sqrt{HH-BB}$.

Case VIII. *Given the Sum of the* Base *and Greater* Segment *of the* Hypothenuse, viz. $B+A=S$; *and the Sum of the* Cathetus *and Lesser* Segment, viz. $C+D=X$; *thence to find the* Sides *and* Segments *severally.*



Put

Put $2z=\frac{XX}{2S}+2X-\frac{1}{2}S.$

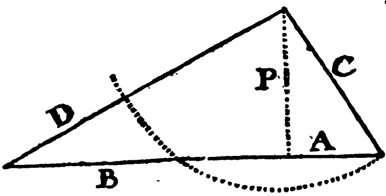
1 Theorem. $A=\sqrt{zS+zz}-z.$

2 Theorem. $B=S-A.$

3 Theorem. $D=\frac{SS}{A}-2S.$

4 Theorem. $C=X-D.$

Case IX. *Given (in any* Plain Triangle) *the Di-ference of the* Sides D *the Difference of t[he] Segments of the* Ba[se] B; *and the* Perpen-dicular (*let fall fro[m] the vertical* Angle) P *thence to find all th[e]* Sides.



Put $2z=BB-DD.$

1 Theorem. $A=\sqrt{\frac{1}{4}BB+\frac{PPDD}{2z}-2z}-\frac{1}{2}B.$

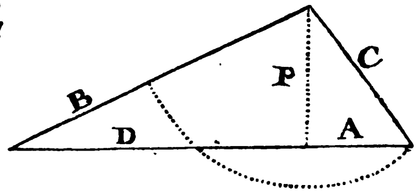
2 Theorem. $B+2A=$ The Base.

3 Theorem. $C=\frac{z+BA}{D}=$ Lesser Side.

4 Theorem. $C+D=$ The Greater Side.

Ca

Cafe X. *The Sum of the two Sides of any* Plain Triangle S; *the Difference of the* Segment *of the* Bafe D; *and the* Perpendicular P (*let fall from the* Vertical-Angle) *being Given*; *thence to find the* Bafe, *and the* Sides.



Suppofe $2z = SS - DD$

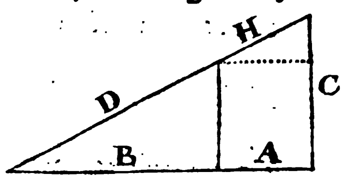
1 Theorem. $A = \sqrt{\frac{1}{4}DD + \frac{1}{2}z - \frac{SSPP}{2z}} - \frac{1}{2}D.$

2 Theorem, $C = \frac{z - DA}{S}$ The Leffer Side.

3 Theorem. $D + 2A =$ The Bafe.

4 Theorem. $S - C = B + C =$ Greater Side.

Cafe XI. *In any* Right-angled Triangle, *let a* Right Line *be drawn parallel to the* Cathetus, *and let there be Given the* Cathetus C; *that* Segment *of the* Hypothenufe, *next to the* Cathetus H; *and the alternate* Segment *of the* Bafe B; *to find the* Bafe *and* Hypothenufe.

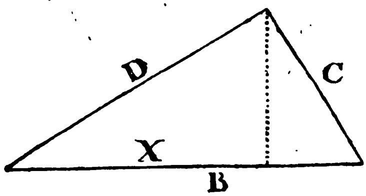


Theorem

$$A^4 + 2BA^3 + C^2A^2 + B^2A^2 - H^2A^2 - 2H^2BA = H^2B^2$$

Then $A + B =$ The Bafe; whence the *Hypothenufe* is known by the laft *Method.*

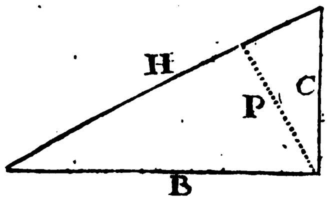
Cafe



Case XII. *Given the* Sides B, C, *and* D *of any* Plain Triangle; *to find the Greater Segment of the* Base X.

Theorem.

$$X=\frac{BB+CC+DD}{2B}$$



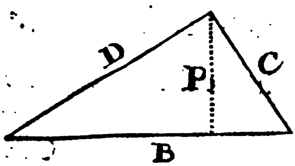
Case XIII. *Given the Perimeter of a* Right-angled Triangle S, *and the* Perpendicular *let fall from the* Right-Angle P; *to find the* Sides.

The Perimiter $B+C+H=S$.

1 Theorem. $H=\frac{SS}{2S+2P}$

2 Theorem. $B+C=S-H$, the Sum of the Sides

3 Theorem. $B-C=\sqrt{HH+2SH-SS}$, The Difference of the Sides.



Case XIV. *Given the* Base B; *and the Sum of the* Perpendicular *and the* Legs (viz. $P+D+C=$) S; *to find the Parts severally, of any* Right-Angle Triangle.

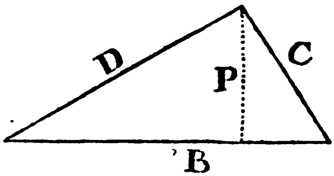
1 Theorem

1 Theorem. $P = S + B - \sqrt{2SB + 2BB}$

2 Theorem. $D + C = S - P$, the Sum of the Sides.

3 Theorem. $D - C = \sqrt{2BB - SS + 2SP - PP} =$ The Difference of the Sides.

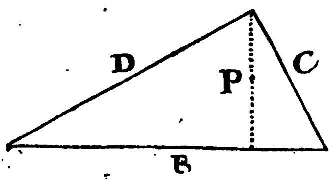
Case XV. *Having Given in a* Right-angled Triangle *the Sum of the* Sides $(D + C =)$ S; *and the* Perpendicular P; *to find the* Triangle.



1 Theorem. $B = \sqrt{PP + SS} - P$.

2 Theorem. $D - C = \sqrt{2BB - SS}$.

Case XVI. *Having Given in a* Right-angled Triangle, *the Sum of the* Legs $(D + C =)$ S; *and the Sum of the* Perpendicular *and* Base $(P + B =)$ X; *to find the* Triangle.

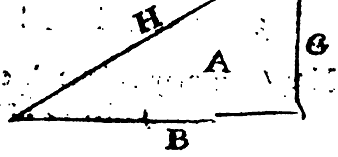


1 Theorem. $D = \frac{1}{2}S \pm \sqrt{XX - \frac{3}{4}SS - X\sqrt{XX - SS}}$.

2 Theorem. $C = S - D =$ The Cathetus.

3 Theorem. $B = \sqrt{DD + CC}$, the Hypothenuse.

Case XVII. *The Perimeter, or Sum of the three* Sides *of a* Right-angled Triangle *Given* $(B + C + H =)$ S; *and the* Area *thereof* A; *thence to find each* Side.

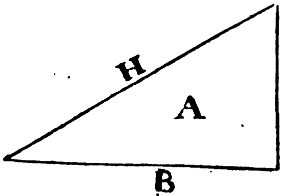


1 Theorem. $H = \frac{1}{2}S - \frac{2A}{S}$.

2 Theorem. $B = S - H + \sqrt{HH - 4A}$.

3 Theorem. $C = S - H - B$.

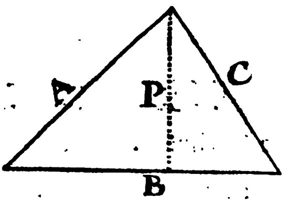
Case XVIII. *In any* Right-angled Triangle, *the* Area A; *and the Sum of the* Hypothenuse *and either* Side (*suppose* H+C=) S; *being Given*; *thence to find the* Sides.



1 Theorem. $BSS - BBB = 4SA$.

2 Theorem. $C = \frac{2A}{B}$.

3 Theorem. $H = S - C$.



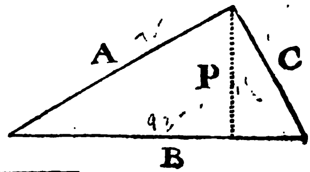
Case XIX. *To find a* Triangle, *whose three* Sides A, B, C; *and* Perpendicular P, *are in* Arithmetical Progression.

1 Theorem. $8C^3 - 36AC^2 + 54AAC = 25A^3$.

2 Theorem. $B = 2A - C$.

3 Theorem. $P = 2C - A$.

Note, You may assume A equal to any Number, in order to find the Rest.



Cafe XX *To find a* Triangle, *whofe three* Sides A, B, C; *and the* Perpendicular P, *are in* Geometrical Progreffion.

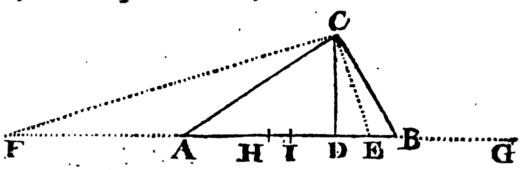
1 Theorem. $A = C\sqrt{\frac{1}{2}+\sqrt{\frac{2}{4}}}$

2 Theorem. $B = \frac{AA}{C}$

3 Theorem. $P = \frac{CC}{A}$

Note, Here again you may affume C = any Number at pleafure, to find the Reft.

Cafe XXI. *Having the* Sides *and* Bafe *of any* Right-lined Triangle *Given*; *to find the Segments of the* Bafe, *the* Perpendicular, *the* Area *and the* Angles.



Bifect AB in I; and make AF and AE=AC; and BG and BH=BC; join CE and CF; and let fall the *Perpendicular* CD, on the *Bafe* AB. Then it will be,

Theorem 1. for the Segments of the *Bafe*. $\frac{ACq - BCq}{2AB} = DI$ Now BI—DI =DB, the leffer Segment.

Theorem 2. for the *Perpendicular*. $\frac{\sqrt{FG \times FH \times HE \times EG}}{2AB} = CD$ The *Perpendicular*

Theorem 3. for *Area*. $\frac{\sqrt{FG \times FH \times HE \times EG}}{4} =$ The *Area*.

 For

For the *Angle* A, there are the several *Theorems* following.

1. As $2AB \times AC : HE \times EG$ (: :AC : DE) : : Radius : versed Sine of A.

2. As $2AB \times AC : \sqrt{FG \times FH}$ (: :AC : FD) : : Radius : versed Co-sine of A.

3. As $2AB \times AC : \sqrt{FG \times FH \times HE \times EG}$ (: :AC : CD) : : Radius : Sine of A.

4. As $\sqrt{FG \times FH} : \sqrt{HE \times EG}$ (: : CF : CE) : : Radius : Tangent of $\frac{1}{2}$ A.

5. As $\sqrt{HE \times EG} : \sqrt{FG \times FH}$ (: :CE : CF) : : Radius : Co-Tangent of $\frac{1}{2}$ A.

6. As $2\sqrt{AB \times AC} : \sqrt{HE \times EG}$ (: :FE : EC) : : Radius : Sine of $\frac{1}{2}$ A.

7. As $2\sqrt{AB \times AC} : \sqrt{FG \times FH}$ (: : FE : FC) : : Radius : Co-Sine of $\frac{1}{2}$ A.

Thus I have furnished the *Young Algebraical* (the only true) *Geometer*, with a Collection of the most Useful and Curious of all the *Anomalous Cases of Plain Trigonometry*, with the *Theorems* resolving the same; I had Thoughts of adding more, but considering 'twas to little Purpose, I chuse rather to refer to the Learned Authors, Mr. *Ward*, Mr. *Rapson*, Dr. *Harris*, Sir *Isaac Newton*, &c. whence these were taken; and shall now proceed to give an *Example* of the *Method* of Investigating or Raising the foregoing *Theorems*; and shall shew the Reason of the Process, by an Apposition of the several *Axioms* and *Geometrical Theorems*, which are the Rules and Foundation of the whole Matter. The

The *Example* ſhall be of *Caſe* IV. for the firſt *Theorem*, in order to find the *Baſe* B; and it is thus.

Let	1	$H=194$ } For theſe are Parts in that Caſe given.
And	2	$D=14=B-C$
Then	3	$BB+CC=HH=37636$. by *The.* XI.
2 ſquared	4	$BB-2BC+CC=DD=196$. by *Ax.* 6.
3 leſs 4	5	$2BC=HH-DD=37440$. by *Ax.* 2.
3 more 5	6	$BB+2BC+CC=2HH-DD=75076$. (by *Ax.* 1.
6 extracted	7	$B+C=\sqrt{2HH-DD}=274$. by *Ax.* 6.
2 more 7	8	$2B=D+\sqrt{2HH-DD}=288$. by *Ax.* 1.
8 divid. by 2	9	$B=\frac{D+\sqrt{2HH-DD}}{2}=144$. by *Ax.* 4.
7 leſs 2	10	$2C=\sqrt{2HH-DD}-D=260$. by *Ax.* 2.
10 divid. by 2	11	$C=\frac{\sqrt{2HH-DD}-D}{2}=130$. by *Ax.* 4.

Thus by the above *Proceſs* at the 9th Step, there comes out the *Firſt Theorem*, which ſhews the *Baſe* to be 144; and by the 11th Step, you have the *Second Theorem*, ſhewing the *Cathetus* to be 130; and after this Manner are the other *Theorems* inveſtigated, and reſolved into Numbers; which I leave to exerciſe the young Artiſt when it ſuits his Occaſion; and proceed to the next *Chapter* of *Oblique-angled Plain Triangles*.

CHAP.

CHAP. XVI.

Of the Solution of Oblique-angled Plain Triangles.

WHAT the Species of *Oblique Triangles* are may be ſeen in the *Definitions*; you will there find them to be of two Sorts, *viz.* ſuch as have all their Angles Acute, or each leſs than 90°; or ſuch as have one Angle Obtuſe, *viz.* greater than 90°. For all the Angles may be ſeverally leſs than a Right Angle; but one Angle only can be bigger; by *Theorem* IV.

Every *Oblique-angled Triangle* may be ſolved, if there be Given therein the following Parts, *viz.*

First, The three Angles, and any one Side.

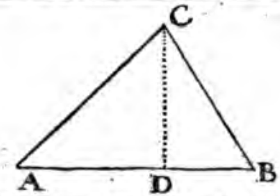
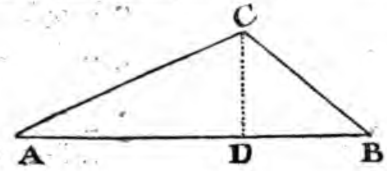
Secondly, Two Sides, and an Angle oppoſite to one of them.

Thirdly, Two Sides, and an Angle comprehended between them.

Fourthly, All the three Sides.

And theſe four *Caſes* are all that can happen in an *Oblique-angled Plain Triangle*; Notwithſtanding which, moſt of our principal Trigonometrical Writers make, ſome five, and ſome ſix *Caſes*; but without any Reaſon; for 'tis the ſame *Caſe* to find all the unknown Parts, as to find one of them, by the ſame *Data*. Alſo, to have the three Angles only Given, is not properly any *Caſe* at all; for by them the Proportion of the Sides, but not the Sides themſelves, can only be found; therefore there can be but four *Caſes*; *A Synopſis* which here follows.

Caſes

Cases.	Given	Required.	 
I.	AB, C, AB.	AC, CB.	As sC : AB : : sB : AC. As sB : AC : : sA : BC. } By *Theorem* XXVIII.
II.	AC, BC, A.	AB, CB,	As BC : sA : : AC : sB; Then, 180°−A−B=C. As sB : AC : : sC : AB; as before.
III.	AC, BC, C.	A, B, AB.	As AC+CB : AC−CB : : $t\frac{A+B}{2} : t\frac{A-B}{2}$. By *Theorem* XXIX. Hence the Angles A and B are known, by *Theorem* Then, As sA : BC : : sC : AB, as before.
IV.	AB, AC, CB.	A, B, C.	As AB : AC+CB : : AC−CB : AD−DB, by *Theorem* XXX. Hence the Segments AD and DB, are known by *Theorem* And the Angles A, B and C, are found by Resolving the two Right-angled Triangles ADC and BDC.

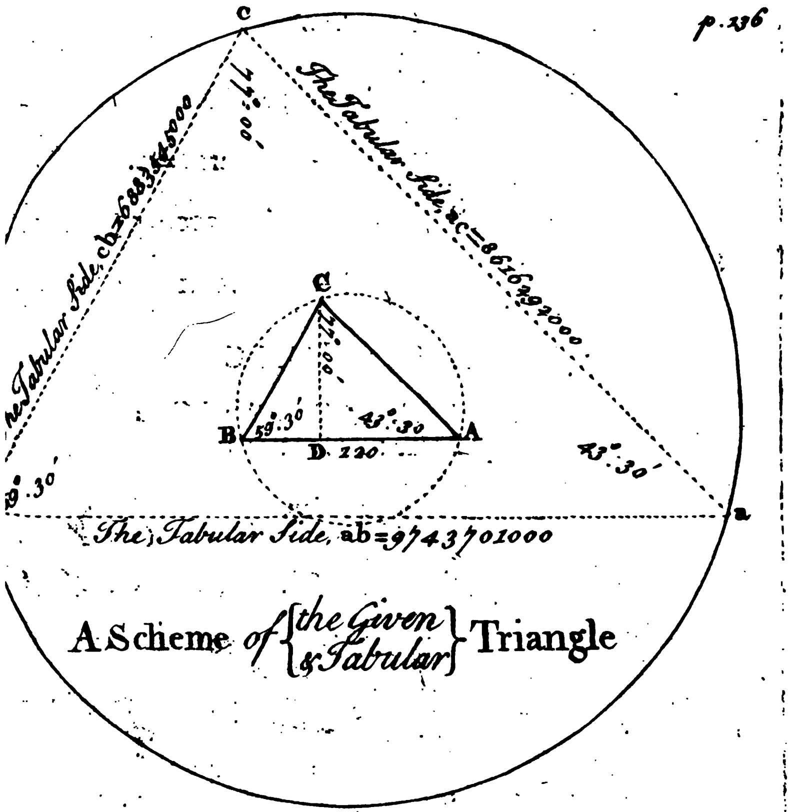
In

In each *Case* I have referred to the *Theorems* wherein it is demonstrated, which you would do well to Consult, or rather to make your self perfectly acquainted with them; that so you may never be at a loss how to proceed directly in any kind of *Trigonometrical Calculation.* And as I have largely shewn how to solve all the *Cases of Right-angled Plain Triangles* by all the best *Methods* in Use, so it is not necessary here to repeat them; and therefore I shall only give a *Numerical Solution* of these four *Cases of Oblique Triangles* by one of the best *Methods*, viz. by *Artificial Sines* and *Tangents*, or by *Logarithms.*

But as I have done before, so here I have given a Scheme of the *Given Triangle*, and That in the Tables, represented by *Numbers*: The *Given Triangle* is ACB, similar to which, and similarly circumscribed is the *Tabular Triangle* acb. For because by *Theorem* XV. the Sides which are alike in *Similar Triangles* are in Proportion; and also by *Theor.* XXVIII. the Sides of *Plain Triangles* are to each other, as the Sines of their opposite *Angles*; Therefore, As AB : BC (: : s C : s A) : : ab : bc; and so of the other *Sides* and *Angles.*

Case I.

p. 136



A Scheme of {the Given & Tabular} Triangle

Case I. *Given the* Angle A=43° 30'; B=59° 30'; C=77° 30', *and the* Side AB=120; *To find the* Sides AC *and* BC.

The *Analogies* for the Side AC. { As sC : AB : : sB : AC, in the *Synopsis*, *i.e.* As ab : AB : : ac : AC, in the *Scheme*.

T *Operation*.

Operation.

		Com. Arith.
As the Tabular Side (or Sine of 77° 00′)	ab=9743701000	=0.0112761
Is to the Side of the Given Triangle,	AB= 120	=2.0791812
So is the Tabular Side (or Sine of 59° 30′)	ac=8616292000	=9.9353204
To the Side sought of the given Trian.	AC= 106.1	=2.0257777

To find the Side BC.

The *Analogies* { As sB : AC : : sA : BC. That is, As ac : AC : : bc : BC, in the *Scheme.*

Operation.

		Com. Arith.
As the Tabular Side (or Sine of 59° 30′)	ac=8616292000	=0.0646796
Is to the Side of the Given Triangle——	AC= 106.1	=2.0257777
So is the Tabular Side (or Sine of 43° 30′)	bc=6883545000	=9.8378122
To the Side sought of the Given Triangle	BC= 84.77	=1.9282695

Case II. *Two* Sides, AC=106.1, *and* BC=84.77, *and an* Angle A=43° 30′, *opposite to one of them, being Given*; *thence to find the other* Angles *and* Sides.

Note; As this *Case* is but the Reverse of the last, it needs no *Example*; only you must observe, that if one of the two Sides given be the Greatest Side, and the Angle opposite to it be sought, the Angle when found will be ambiguous, that is, 'twill not be certain whether

whether it be Accute or Obtuse; because the Sine of an Arch, and the Sine of that Arch's Complement to 180 Degrees is all one, as I have before shewn; and therefore, in Matters of Consequence, 'twill be best to delineate the *Triangle* by *Method* VI. or by calculating the third *Angle*, in order to resolve the Ambiguity.

Case III. *Two* Sides AC=106.1, *and* BC=84.77, *and the* Angle *included* C=77° 00′, *being Given*; *thence to find the other Parts.*

Analogy for the Angles: As AC+CB : AC−CB :: $t\frac{A+B}{2} : t\frac{A-B}{2}$.

Operation.

Add	The Side	AC=106.1
	The Side	CB= 84.77
The Sum of the two Sides		AC+CB=190.87
The Difference of the Sides		AC−CB= 21.33
The ½ Sum of the two Angles		$\frac{A+B}{2}$= 51° 30′.

		Com. Arith.
Therefore, As the Sum of the two Sides	190.87	=7.7192396
Is to their Difference	21.33	=1.3289909
So is the Tangent of half the Sum of the unknown Angles	51° 30′	=10.0993948
To the Tangent of half their Difference,	$\frac{A-B}{2}$=8° 0′	=9.1476253

Therefore to half the Sum of the Angles 51° 30′
Add half the Difference of those Angles— 8 00

The Sum is the Greater Angle——B= 59 30

The Difference is the Lesser Angle—A= 43 30

And now the Angles being all known, the other Side AB is to be found by the common *Analogy* of *Case* II.

Case IV. *Given all three* Sides, AB=120; AC=106.1; BC=84.77; *to find the* Angles.

The *Analogy* for the Segments of the *Base* AD, BD.
As AB : AC+CB : : AC−CB : AD−DB.

Operation.

		Com. Arith.
As the Greater Side or Base	AB=120=	7.9208188
Is to the Sum of the other two Sides	AC+CB=190.87=	2.2807604
So is the Difference of those Sides —	AC−CB= 21.33=	1.3289909
To the Diff. of the Seg.	AD−DB=26.95=	1.4305701

Hence, to ½ the Sum of the Seg. $\frac{AD+DB}{2}=60.$

Add half the Differ. now found, $\frac{AD-DB}{2}=13.47$ &c.

The Sum is the Greater Segment AD=73.47 &c.

The Difference is the Lesser Segment DB=46.53

Hence

Hence is the *Oblique Triangle* ABC resolved into the two *Right-angled Triangles* ADC and BDC, in each of which there is the *Base* and *Hypothenuse* Given; consequently the two *Angles* A and B are found by *Case* V. of *Right-angled Triangles* aforegoing. Thus are all the Cases of *Oblique-angled Triangles* to be resolved: But there is another Way yet to find the *Angles* in this last *Case* by one *Operation*, and is as follows.

Another Way to resolve Case IV.

Add together the three Given Sides $\begin{cases} AC=106.1 \\ AB=120 \\ BC=\ 84.77 \end{cases}$

Their Sum is ————————	310.87
The half Sum is ————————	155.43

From which subtract severally the two Sides AC and BC, including the Angle sought C; and their Differences will be ——— $\begin{cases} 49.33 = \text{Diff. AC.} \\ 70.66 = \text{Diff. BC.} \end{cases}$

Then to the Arithmetical Complements of the Sides $\begin{cases} AC=106.1 \\ BC=84.77 \end{cases}$	$\begin{matrix} =7.9472846 \\ =8.0717578 \end{matrix}$
Add the Logarithms of the Differences before found $\begin{cases} AC=49.33 \\ BC=70.66 \end{cases}$	$\begin{matrix} =1.6931111 \\ =1.8491736 \end{matrix}$
The Sum of all which is ————	19.5883271
The half whereof is the Sine of half the sought Angle C ——	$38^\circ\ 30' = 9.7941635$

Double of which is the Angle C = 77° 00′, as required.

Whence the other Angles are found by *Case* I.

There

There are yet other Ways to solve this fourth *Case*; one of which for finding the Segments of the Base, is *Case* XII of *Method* IX. of solving *Plain Triangles* by *Algebra*.

I had Thoughts of adding several *Anomalous Cases* of *Oblique Triangles*, with *Theorems* for their Solution; but I shall omit that, because I have already partly done it by the foregoing ninth *Method of solving Right-angled Triangles*, several of the *Cases* there being common also *Oblique Triangles*; and also because such *Cases* do not very often casually occur, and therefore though one should give a Collection of them, it might be to little Purpose; and perhaps, when a *Case* of that Nature happens, it may not be found among them; and lastly, considering that none but the *Algebraists* are sufficient for such Matters, and they not needing any such Supplies, I shall wave it; and pass to the next *Chapter*.

CHAP. XVII.

Of the Dimensition of the Superficies of a Right and Oblique Plain Triangle; or how to find its Area by certain Sides and Angles given.

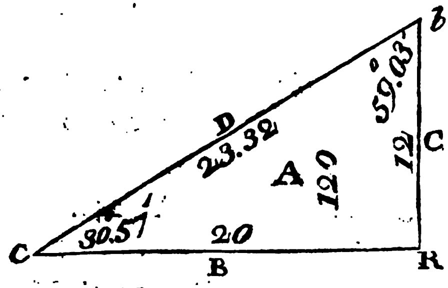
IT is a Matter generally of no great Moment or Difficulty, to be able to measure the Superficial Content or Area of any *Plain Triangle*, in one certain Manner, *viz.* by having Given the Base, and Height

Height, or Perpendicular: But if any other Parts of the Triangle be Given, and either the Base, or Perpendicular, or both, be unknown; then it is not so easy a Thing to assign the Area with Geometrical Truth, by any common Mechanic or Surveyor.

I have therefore designed this *Chapter* to instruct the *Young Geometer* how to find the Area of any *Plain Triangle*, by any sufficient *Data* of Sides, or Sides and Angles mixtly; and for that Purpose I have exhibited *Theorems* for the true understanding and expeditious Operation of every different *Case* of the *Data*; which *Theorems* I have not found Given by any other Author, excepting the First only.

And those for a *Right-angled Triangle* are as follows.

In the following *Triangle*, let B = Base; C = Cathetus; D = Hypothenuse; and let c and b represent the Sines of the Angles at which they stand; and R = Radius, or Sine of the Right-Angle; and A = the Area.



Case I. *Given the Base* B = 20, *and the* Cathetus C = 12; *to find the Area* = A?

$$\text{Theorem. } \frac{BC}{2} = A = 120.$$

Case II.

Case II. *Given the* Base B=20; *and* Hypothenuse D=23.32; *to find* A= *the* Area?

Theorem. $B\sqrt{\overline{D+B}\times\overline{D-B}}=2A=240.$

Case III. *Given the* Cathetus C=12; *and the* Hypothenuse D=23.32; *to find the* Area A?

Theorem. $C\sqrt{\overline{D+C}\times\overline{D-C}}=2A=240.$

Case IV. *Given the* Base B=20; *and the* Angles c=30° 57′, *and* b=59° 03′; *to find the* Area = A?

Theorem. $\frac{BBc}{b}=2A=240,$

Case V. *Given the* Cathetus C=12; *and the* Angles c=30° 57′; *and* b=59° 03; *to find* A = Area?

Theorem. $\frac{CCb}{c}=2A=240.$

Case VI. *Given the* Hypothenuse D=23.32; *and the* Angles c=30° 57′; *and* b=59° 03′; *to find the* Area = A?

Theorem. $\frac{DDbc}{RR}=2A=240.$

Case VII. *Given the* Base B=20; *the* Hypothenuse D=23.32; *and* Angle *at* Base c=30° 57′; *to find* A?

Theorem. $\frac{DBc}{R}=2A=240,$

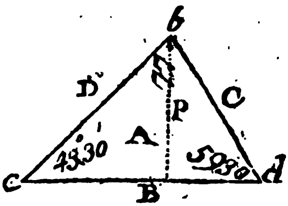
Case VIII.

Case VIII. *Given the* Cathetus C=12; *and* Hypothenuse D=23.32; *and* Angle *at* Perpendicular b=59° 03′; *to find the* Area=A?

Theorem. $\frac{DCb}{R}=2A=240.$

Thus by having any two Sides, or any one Side and the Angles Given, in any Right-angled Triangle, the Area is found with ease by the foregoing *Theorems*.

Now follow the *Theorems* in like Manner for finding the Area of an *Oblique-angled Triangle*.



Case I. *Given in the adjacent* Oblique Triangle, *the* Angles c=43° 30′; d=59° 30′; *and* b=77°; *and the* Side B=12; *to find the* Area=A?

Theorem. $\frac{BBcd}{Rb}=2A.$

Case II. *Given two* Sides D=10.61; *and* C=8.47; *and the* Angle *included*, b=77° 00′; *to find the* Area=A?

Theorem. $\frac{DCb}{R}=2A.$

Case III. *Given the three* Sides B=12; D=10.61; *and* C=8.47; *thence to find the* Area =A?

See the *Theorem* for this *Case*, in *Case* XXI. of the *Ninth Method of Solving Plain Triangles by Algebra.*

The first *Theorem* for the *Right-angled Triangle*, is common to all *Plain Triangles*; and therefore $\frac{BP}{2}=A$ the *Area* of the *Oblique Triangle* here.

The Reason of this common *Theorem* is from the Geometrical *Theorem* IX. where 'tis shewn, that a *Triangle* is just half its circumscribing *Parallelogram*; and therefore as the Base and Height of any *Plain Triangle* and its *Parallelogram* are the same; and the Product of these being the *Area* of the *Parallelogram*; it follows, that half that Product is the *Area* of the *Triangle*.

I shall illustrate this Affair by exemplifying *Case* I, II, and VI, of the *Right Triangle*; and *Case* III of the *Oblique One*, by *Logarithms*.

Case I. Theorem. $\frac{BC}{2}=A$.

To the Logarithm of the Base B=20=1.3010300
Add the Logar. of the Cathetus C=12=1.0791812

The Sum is the Logarithm of BC=240=2.3802112

Which divided by 2, is —— $\frac{BC}{2}=120=A$ the *Area*

Case II.

Caſe II. Theorem. $B\sqrt{D+B\times D-B}=2A$.

To the Logar. of the Sum of the Baſe and Hypot. } $D+B=43.32=1.6366884$
Add the Logar. of their Difference ———— } $D-B=3.32=9.5211381$
The Sum is ——— $\overline{D+B\times D-B}$ $=2.1578265$

The half Sum is —— $\sqrt{D+B\times D-B}$ $=1.0789132$
To that add the Logar. of the Baſe $B=20=1.3010300$
The Sum, is the Logarithm of the Double Area, —— } $2A=240=2.3799432$
as before.

Caſe VI. Theorem. $\frac{DDbc}{RR}=2A$.

The Logar. of the Hypoth. $D=23.32=1.3677285$
The ſame again ————— 1.3677285
Add the Sine of the Ang. at Baſe $c=30°\ 57'=9.7112080$
And the Sine of the Ang. at Perp. $b=59\ 03=9.9332931$

The Sum of all is the Logar. of $DDbc=21.3799581$
From which ſubtract Double Logarithm of Radius, —— } $=RR=20.0000000$
There remains the Logarithm of the Double Area, —— } $2A=240=1.3799581$
as before.

Caſe III. *Of the* Oblique Triangle *reſolved.*

Theorem. $\frac{1}{4}\sqrt{FG\times FH\times HE\times EG}=A$.

From a due Conſideration of the Structure of the *Scheme* to *Caſe* XXI. of *Method* IX. You may obſerve the ſeveral Parts of this *Theorem*, to anſwer to the Sides, *&c.* of this *Oblique Triangle* as is here ſet down, *viz.*

FG=B+C+D, the ſum of the three Sides, *viz.* 31.08
FH=B+D—C=12+10.16—8.47 = ——— 14.14
HE=D+C—B=10.61+8.47—12 = ——— 7.08
EG=B+C—D=12+8.47—10.61 = ——— 9.86

Therefore the *Operation* is very eaſy by *Logarithms*, thus;

Add the Logarithms of— { FG=31.08=1.4924810, FH=14.14=1.1504494, HE= 7.08=0.8500333, EG= 9.86=0.9938769 }

The Sum of all is ——————— 4.4868406

½ Sum is the Logar. of the Squ. Root 175.1=2.2434203
Subduct the Logarithm of ——— 4=0.6020600

Remains the Log. of the Area, A=43.78=1.6413603

Thus I have ſhewn a *Method* very General, and New (as to the Form) for finding the Area's of all *Plain Triangles* with great Exactneſs and Expedition. They who would ſee the Demonſtration of this laſt excellent *Theorem* (or rather its Analytic Inveſtigation) may be ſatisfied by conſulting *Problem* XXIII and LI. of Sir *Iſaac Newton*'s *Algebra*.

The

The Second Part:

CONTAINING,

The Application of *Plain Trigonometry* to the Ten following Mathematical Arts and Sciences.

Viz.

I. *Navigation* in Six ſeveral Kinds.
II. *Coſmography* and *Geography*.
III. *Aſtronomy*.
IV. *Fortification*.
V. *The Doctrine* of *Projectiles*, or *Gunnery*.
VI. *Mechanics*, or *Science* of *Motion*.
VII. *Altimetry* and *Longimetry*.
VIII. *Surveying*, &c.
IX. *Optics* in both its Branches.
X. *Perſpective*.

In all which are many Propoſitions very rare and curious; and ſeveral not any where elſe to be found. The Whole being a more compleat Application of this Art, than was ever yet extant.

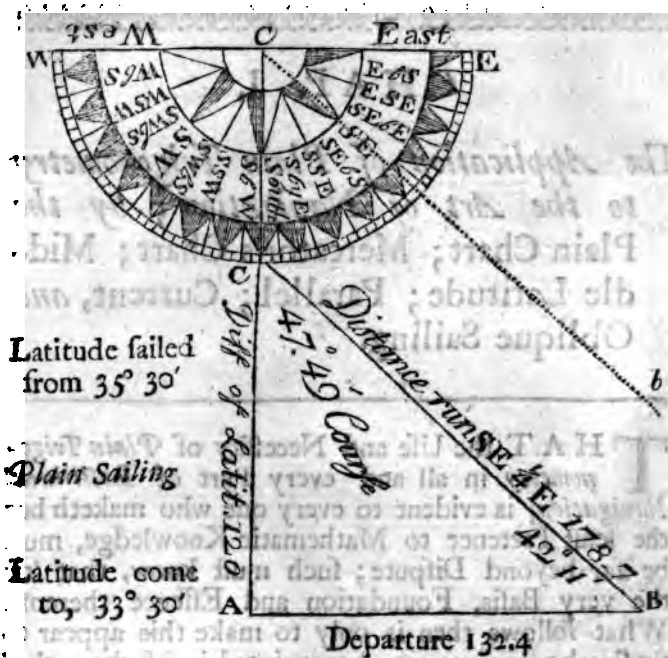
CHAP. I.

The Application of Plain Trigonometry to the Art of Navigation; by the Plain Chart; Mercator's Chart; Middle Latitude; Parallel; Current, *and* Oblique Sailing.

THAT the Use and Necessity of *Plain Trigonometry* in all and every Part of *Rectilineal Navigation*, is evident to every one who maketh but the least Pretence to Mathematic Knowledge, must be far beyond Dispute; such must know, that 'tis the very Basis, Foundation and Essence thereof: What follows then is only to make this appear to those who are as yet unacquainted in Mathematical Studies, and have a desire to be informed therein; and may notwithstanding serve as an *Amanuensis* even to the Speculative Mathematician; who for want of constant-Practice, must stand in need sometimes of making a cursory Review or Perusal of the various Parts of this Art.

I shall begin with shewing how a *Right-angled Plain Triangle* is applied to Sailing by the *Plain Chart*, called *Plain Sailing*.

Latitude



In the Figure above, WCE represents one half of the *Sea-Compass*, divided into its proper Points or Winds, assigned by their proper Names.

Suppose a Ship in the Latitude of 35° 30′ North, sets sail between the South and East Points, suppose 4¼ Points to the East, or on the Rumb SE ¼ E. which makes an Angle with the Meridian or South Point, of 47° 49′; and she saileth 'till her Difference of Latitude be just 2°, or 120 Miles, or 40 Leagues; and it be required to find her Distance run, and Departure from her first Meridian, or Difference of Longitude.

In this Case, let C represent the Ships first Place in Latitude 35° 30′, then continue the Meridian Line

of

of the Compaſs c C to A, and make CA=120 Miles; and now becauſe, in the Compaſs, the dotted Line b c is the Rumb on which the Ship ſails, and b c C the Angle of the Courſe; therefore if from the Point C there be drawn the Line CB parallel to c b, the Angle BCA ſhall be equal to the Angle b c C (or b c A,) by *Theorem* III; and conſequently the Angle C is the Ship's Courſe, and the Line CB the Rumb on which ſhe runs her Diſtance; on A draw the Line AB at Right Angles, and continue it 'till it meet the Rumb CB in B; then is the Line BA the Departure, or Difference of Longitude the Ship has made in ſailing from C to B.

From this Conſtruction 'tis evident there ariſeth a Right-angled Triangle, *viz.* The Triangle ACB Right-angled at A; in which,

1. The *Cathetus* AC, is the Difference of Latitude.
2. The *Baſe* AB, is the Departure; or Difference of Longitude.
3. The *Hypothenuſe* CB, is the Rumb on which the Diſtance is ſailed.
4. The *Angle at Perpendicular* ACB, is the Ship's Courſe. And
5. The *Angle at Baſe* ABC, is the Complement of the Courſe.

From all this 'tis manifeſt, that the Solution of Caſes of Sailing by the *Plain Chart*, is nothing elſe but the Reſolving the ſeveral Caſes of a *Right-angled Plain Triangle* by any of the foregoing *Methods*.

Example of all the Six Caſes.

I. *Given the Ships Difference of Latitude* AC=120 Miles; *and Courſe* SE$\frac{1}{4}$ E, *or the Angle* C=47° 49'.

X Then

Then by *Case* II, of *Right Triangles*, the Distance run BC will be found to be 178.7 Miles; and the Departure AB=132.4 Miles, or 2° 12′ Difference of Longitude.

II. *Given the Ship's Course* C, *and Distance* CB, *as before found; Then will the Difference of Latitude* CA, *and Departure* AB, *be found, by* Case III *of* Right Triangles.

III. *Given the Course* C, *and Departure* AB; *then is the Distance run* CB, *and Difference of Latitude* AC, *found by* Case I *of* Right Triangles.

IV. *Given the Distance run* CB, *and Difference of Latitude* AC; *then shall the Course* C *and Departure* AB, *be found by* Case VI *of* Right Triangles.

V. *Given the Difference of Latitude* AC, *and Departure* AB; *then the Course* C, *and Distance sailed* CB, *is found by* Case IV *of* Right Triangles.

VI. *Given the Distance sailed* CB, *and the Departure* AB; *then will the Difference of Latitude* CA, *and the Course* C, *be found by* Case V *of* Right-angled Triangles.

And these are all the *Cases of Plain Sailing*; which cannot need the *Operations in Numbers* to make it more evident and intelligible than it is already in every Part, to any one who understands the foregoing Doctrine of *Plain Right-angled Triangles.*

In this *Chart* the Degrees of Longitude are supposed equal to the Degrees of Latitude in any Place the Ship is in; and therefore as the Degrees of La-titude

titude are represented in Right-Lines, so are the Degrees of Longitude; and hence the Departure and Difference of Longitude, in this kind of Sailing, are all one Thing; but this Supposition is intirely false; and the whole *Chart* founded thereon is erroneous; for the Degrees of Latitude are equal to the Degrees of Longitude only in the *Equator*, but in no other Parallel of Latitude whatever; and therefore this *Chart* can no where be exact, and is of use only in these Parts which lie near the *Equator*, and in *Coast Sailing*.

Mercator's Sailing.

IN order to have a pretty good Notion of *Mercacator Sailing*, or *Navigation* by *Mercator's Chart*, the following Things are to be well attended, on which this Sort of Sailing wholly depends, *viz.*

First, If it be proposed to delineate the Parallels and Rumb-Lines, as well as the Meridians, on a Plane, (with Exactness) in Right-Lines; then because the Degrees on the Parallels decrease in the Proportion of the Co-sine of the Latitude of the Parallel to the Radius, or as Radius to the Secant of the Latitude (as will appear by and by in *Parallel Sailing.*) Therefore in framing this *Chart* the Degrees of Latitude on the Meridians must be inlarged in the same Proportion, toward each Pole; in order that the Degrees of Longitude on the Parallels, may be equal to those on the Equator; and so the Meridians all lie parallel; and the Proportion of Easting and Westing, and of Northing and Southing, may be still the same.

Secondly, 'Tis evident then, that the Length of a Degree, or any other small Arch on the Meridian

thus inlarged, is every where equal to the Secant of its Latitude; For, as Radius 1000 : 60, the nautical Miles in a common Degree of the Meridian, : : 1228 the Secant of 35° 30′ Latitude : 73.68 Miles, an inlarged Degree of the Meridian in that Latitude.

Thirdly, The Diſtance of any Point of the inlarged Meridian from the Equator is equal to the Sum of all the inlarged Degrees or Secants, between it and the Equator.

Fourthly, The Diſtance between any two Parallels on the ſame Side of the Equator is equal to the Difference of the Sums of all the inlarged Degrees or Secants contained between the Equator and each of the Parallels. Thus,

The Sum of the inlarged Degrees to	35° 30′ is	3281
The Sum of the ſame to ———	33° 30′ is	2135
The Difference of theſe Sums ———	is	146

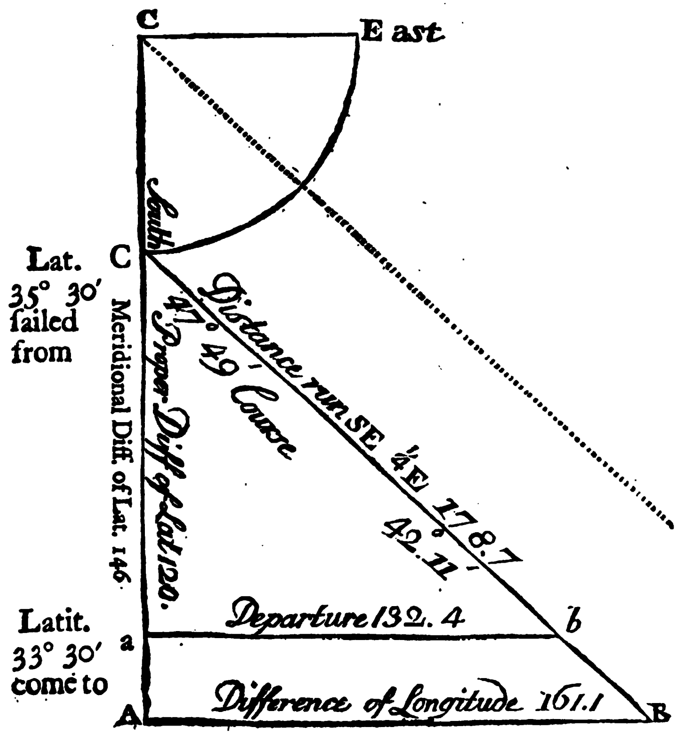
Which therefore is the Diſtance of the two Parallels of 35° 30′, and 33° 30′ Latitude, North or South.

Fifthly, The Diſtance of any two Parallels on contrary Sides of the Equator, is equal to the Sum of the Sums of all the inlarged Degrees or Secants, contained between the Equator and each Parallel.

Sixthly, By the continual Addition of thoſe Secants or inlarged Degrees, is framed a Table, called, *A Table of Meridional Parts*; or (as they may be called) *Mercatorial Miles*; alſo hereby is the Scale of *Meridional Parts* graduated.

Seventhly,

Seventhly, Since in this *Chart* the Meridians are all Parallel Right-Lines; the Rumbs alſo, (whoſe Property is always to make equal Angles with all the Meridians) will here be all ſtrait Lines; conſequently the Ship's Way, Courſe, Difference of Latitude and Longitude, &c. will in this *Chart* alſo form a *Right-angled Triangle*; and if the proper Difference of Latitude and Departure be therein repreſented, there will be conſtituted two *Similar Trangles*; one defective, proper to the *Plain Chart*; the other true and correct, proper to the *Chart*, or Kind of Sailing, now under Conſideration.



Eighthly

Eighthly, Now let the Case before resolved by the *Plain Chart*, be here reasumed; in the preceeding *Scheme* then Ca is the proper Difference of Latitude given; Cb is the Distance run; and ab, the Departure, as in *Plain Sailing*: But that being erroneous, is corrected by this Method; for in the Triangle ACB, there

First, CA is the inlarged Difference of Latitude.
Secondly, AB is the inlarged or real Difference of Longitude.
Thirdly, CB is the inlarged Distance; and
Fourthly, ACB, is the Angle of the Course, as before.

I. *Now in the* Triangle ACB, *there is given* AC= 146, (*the inlarged Difference of Latitude by* Article 4.) *and the Course* SE $\frac{1}{4}$ E. *or* 47° 49′ *the Quantity of the Angle* ACB *to find the Rest.*

Then by *Case* II of *Right-angled Triangles*, the Difference of Longitude AB, will be found 161.1. or 4° 01′. The Departure ab=132.4; and the Distance Cb=178.7. as before.

II. *Given one Latitude, Course* ACB, *and Distance* Cb; *then the proper and inlarged Difference of Latitude* Ca, CA; *and the Departure* ab, *and Difference of Longitude* AB *are found by* Case III *and* II *of* Right Triangles.

III. *Given*

III. *Given the Difference of Latitude* Ca, *and* CA; *and Distance* Cb; *then the Course* aCb, *and Departure* ab, *are found by* Case IV; *and the Difference of Longitude* AB, *by* Case II *of* Right Triangles.

IV. *Given the Difference of Latitude* Ca, CA; *and Difference of Longitude* AB; *then shall the Course* ACB, *and Distance* Cb, *be found by* Case IV; *and the Departure by* Case II *or* III *of* Right Triangles.

V. *Given one Latitude* C, *Course* ACB, *and Difference of Longitude* AB; *then the Difference of Latitude* AC *and* aC, *is found by* Case I; *and the Distance* Cb, *and Departure* ab, *by* Case II *of* Right Triangles.

VI. *Given Difference of* Latitude Ca, CA; *and the Departure* ab; *then the Course* ACB, *and Distance* Cb, *are found by* Case IV; *and the Difference of Longitude* AB, *by* Case II; *or thus*: (by *Theorem* XV.) *as* Ca : CA :: ab : AB. *That is, as the proper Difference of Latitude* : *the inlarged Difference* :: *the Departure* : *the Difference of Longitude.*

These are the principal Cases of *Mercator's Sailing*: This Chart was indeed first published by *Mercator*, and was therefore called by his Name; but was first demonstrated by one Mr. *Edward Wright*, our own Countryman.

In

In this *Diagram* for *Middle Latitude Sailing*,

Ca=FA is the Difference of the two Latitudes given.

ACb=aCb is the Course (as before) SE $\frac{1}{4}$ E. or 47° 49′.

Cb, is the Distance Sailed.

ab, is the Departure; and AB, the Difference of Longitude.

So that the Parts of the Ship's Motion are here found in two *Right-angled Triangles* aCb, and AFB, as in *Mercator Sailing*; only there they were Similar, but here they are not. The Solution then of a *Right Triangle* performs all the Conclusions of *Middle Latitude Sailing*. This kind of Sailing is wholly contained in the two following *Theorems* or *Analogies*:

Viz. $\begin{cases} \text{As FQ : OQ :: FA : AB:} \\ \text{As R : SC :: Cb : ab.} \end{cases}$

The first Analogy *in Words*.

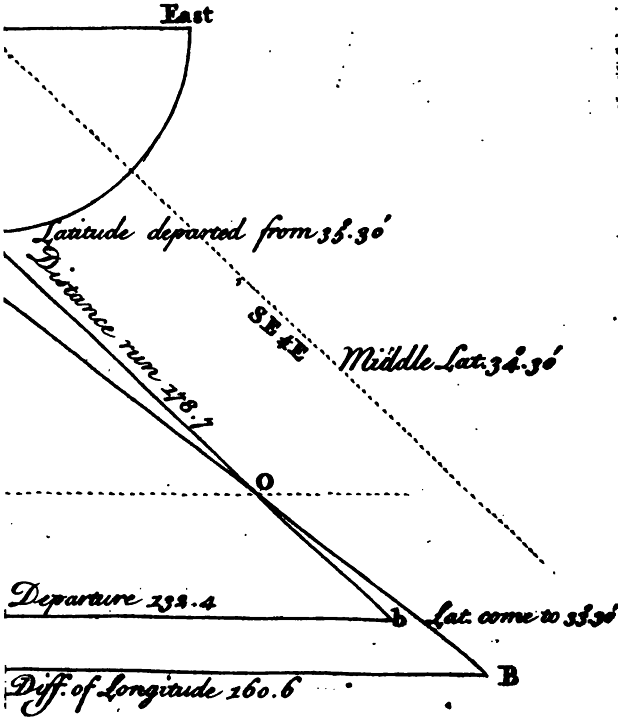
As the Co-Sine of Middle Latitude, FQ
Is to the Tangent of the Course,——OQ
So is the Difference of Latitude ——FA
To the Difference of Longitude ——AB.

The Second Analogy *in Words*.

As Radius R: is to the Sine of the Course C,
So is the Distance Cb, to the Departure ab.

The Demonstration of the first *Analogy* is easy, thus: In the Quadrant ELC, the Middle Latitude is set from E to L; and therefore Le, is the Sine, and eQ the Co-Sine of Middle Latitude; but eQ= FQ; and OQ is the Tangent of the Angle OCQ; viz.

e Latitude Sailing.



viz. the Courſe; farther, the *Triangles* OFQ, and BFA, are ſimilar to each other; and therefore their like Sides will be proportionable, *viz.*

As FQ : OQ : : FA : AB; by Theorem XV.

The Second *Analogy* is only *Caſe* II. of *Right-angled Plain Triangles*.

Suppoſe both the Latitudes given, *viz.* 35° 30′ and 33° 30′, and the Courſe SE ¼ E. or 47° 49′, as before; to find the Difference of Longitude, Diſtance and Departure.

The two Latitudes	35° 30′	— Departed from
	33 30	— Come to
The Sum thereof	69 00	
The half Sum is	34 30	the Mid. Latitude
The Diff. of Latitudes	2 00,	or 120 Miles.

Now to find the Difference of Longitude, ſay;

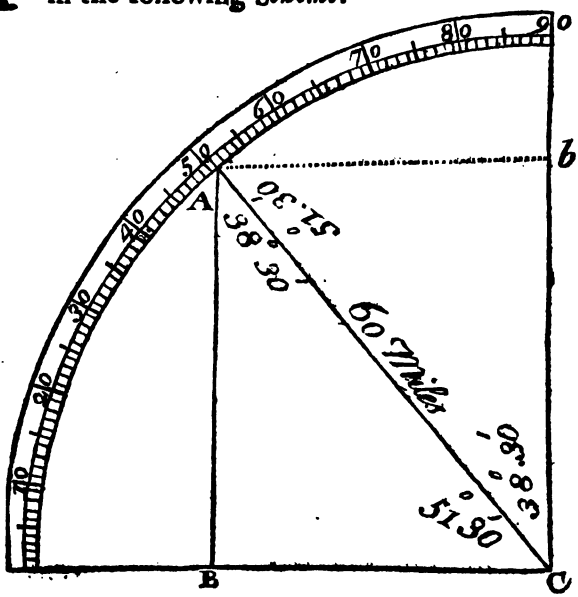
		Com. Arith.
As Co-Sine of Mid. Latitude	—34°30′=	0.0840063
Is to the Tangent of the Courſe	C=47.49 =	10.0427689
So is the Diff. of Latitude FA=120	=	2.0791812
To the Diff. of Longitude AB=160.6=		2.2059564

The Diſtance and Departure, are found as in both the preceeding *Methods* of Sailing, and of the ſame Quantity.

Whatever Caſe ſhall happen is ſpeedily reſolved by the foregoing *Analogies*, and though not ſo preciſely true as by *Mercator's Chart*, yet is ſo very near, as that the Difference is of little Regard; and this *Method* hath this Compenſation, as to be clear and void of all Ambiguity, which *Mercator's Sailing* is not. This is performed without *Tables of Meridional Parts*, with the equal Eaſe of any *Plain Triangle*; wherefore on theſe, and ſeveral other Accounts, this Kind of *Navigation* is deſervedly eſteemed an ingenious Invention, and more naturally exact and expeditious than any other, *Mercator's* not excepted.

Parallel Sailing.

THE *Theory* of this Sort of Sailing is contained in the following *Scheme*.



In

In this *Scheme*, let AB be the Sine of 51° 30′ North Latitude, then ſhall Ab be the Co-Sine of that Latitude, and alſo the Radius of the Parallel thereof; and AC the Radius of a Great Circle of the Sphere.

Of theſe Lines there is compoſed two equal and equiangular Triangles, CAB, and CAb. Make Radius AC=60, the Miles in one Degree of a Great Circle; and there will be Given the Hypothenuſe and the Angles in each: Hence it will be,

I. As the Radius of the Equator AC ——10.0000000
Is to the Co-Sine of the Latitude } A b=BC=38° 30′= 9.7941496
So is the Length of one Degree in the Equator, —— } 60 = 1.7781512
To the Length of one Degree in the Parallel. ——— } 37.35= 1.5723008

II. As the Length of one Degree in the Equator, 60 Miles;
Is to the Length of one Degree in the Parallel of 51°30′, 37.35 Miles;
So is any Difference of Longitude in the Equator, in Miles;
To any Similar Part, or Diſtance Sailed, on the given Parallel.

III. As the Co-Sine of any Parallel, : is to Radius;
So is any Diſtance Sailed on that Parallel,
To the Difference of Longitude on the Equator.

IV. As the Co-Sine of any one Parallel,
Is to the Co-Sine of any other Parallel;
So is the Length of any Arch on the firſt Parallel, in Miles,
To the Length of the ſame Arch on the other, in Miles.

These Proportions evidently flow from the Construction of the *Scheme* above; and from the Proportionality of the Radii of Circles, to the Circles themselves, and also to their Similar Parts; than which scarce any Thing is more commonly known, or more easily proved. And by them are all *the Cases of Parallel Sailing* resolved without any Difficulty, or farther Instruction; my Purpose being only to shew the young Student, how both Theory and Practice of *Navigation* especially, and most other Arts Mathematical, depends on and ariseth from the Nature, Proportion, and Solution of *Plain Triangles.*

Current Sailing.

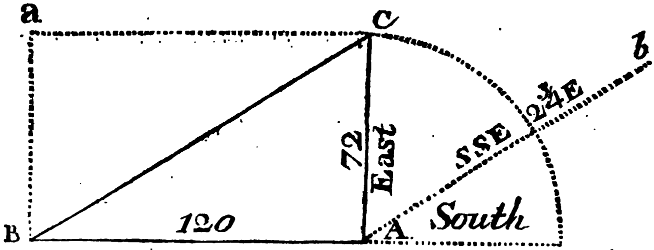
SOME Cases of this Sailing require no *Trigonometry:*

I. *As, suppose a Ship Sails* SE $\frac{1}{4}$E. *at the rate of six Miles an Hour, in a Current that sets* SE $\frac{1}{4}$E. $2\frac{1}{2}$ *Miles an Hour. 'Tis plain the Ships true Motion or Rate of Sailing is the Sum of the Motions both of its Self and of the Current,* viz. $8\frac{1}{2}$ *in an Hour on the same Rumb.*

II. *If a Ship sail* SE $\frac{1}{4}$E. *at the rate of eight Miles an Hour, in a Current that sets* NW $\frac{1}{4}$W. 5 *Miles an Hour. Here 'tis manifest the Difference of the Motion of the Ship and Current (as being directly opposite) will be the true Rate of the Sailing,* viz. *three Miles an Hour.*

III. *Some* Cases *require the Solution of a* Right-angled Triangle; *as suppose a Ship sail directly South* 120 *Miles in* 24 *Hours; and in a Current that sets* 72 *Miles*

Miles East in the same Time, and her Course and Distance be required. In the adjacent Figure, *let* BA *represent the Line of the Ships Course* N *and* S.



when she is at B; *and let* AC *be the East Line of the setting of the Current; then shall the Angle* ABC *be the Ships true Course arising from this compound Motion; and* BC *the Rumb Line, on which she has sailed from* B *to* C.

Hence in the Triangle ABC, *Right-angled* at A, there are Given the Side AB=120 Miles; and AC=72 Miles; to find the Ship's Course ABC; by *Case* IV. of *Right Triangles.*

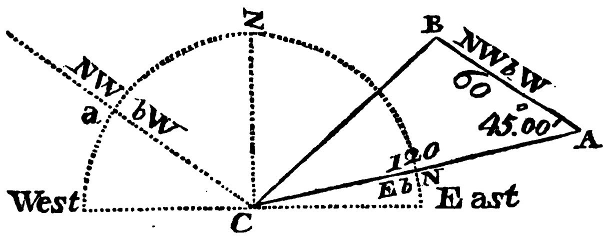
Thus,	As the Ship's proper Motion —	AB=120=	2.0791812
	Is to the Current's Motion. —	AC= 72=	1.8573325
	So is Radius —	—	10.0000000
	To the Tangent of the Ship's true Course —	ABC=30°57′=	9.7781513

Therefore the Ship doth really sail SSE 2¼E; and her Distance sailed, is $\sqrt{ABq+ACq}$=BC=124.8 Miles.

IV. *Some*

IV. *Some* Cases *require the Solution of an* Oblique Triangle; *thus, Suppose a Ship sail* E b N. *at the Rate of* 120 *in* 24 *Hours; in a Current that sets* NWbW. $2\frac{1}{2}$ *Miles an Hour; and her Course and Distance be required.*

In the *Scheme* adjoined, let CA represent the Ship's proper Motion; and AB the Motion of the Current NW b W. (as being parallel to Ca); then shall the



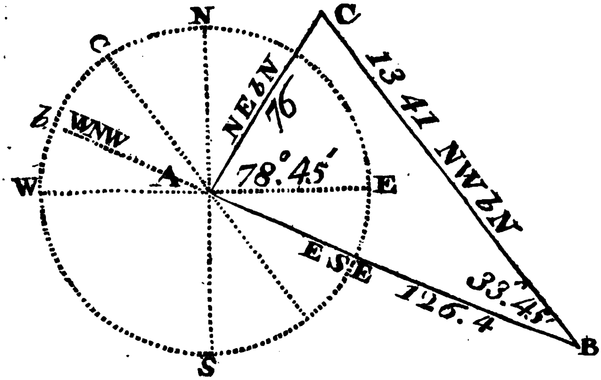
Angle ACB be the Ship's Course; and CB the Rumb or Distance sailed; and B the Place to which the Ship is arrived.

Hence in the *Oblique Triangle* ACB, there is Given two Sides CA=120, and AB=60; and the Angle included CAB=45° 00′, as being equal to four Points; then by *Case* III. of *Oblique Triangles*, there will be found the true Course ACB=33°45′, which added to 11°15′ or EbN. makes 45°, and shews the Rumb CB to be NE. and the Distance sailed thereon from C to B, will be found to be 76.36 Mles; by *Case* I.

Oblique Sailing.

I. *SUPPOSE two Ships sail from the same Port* A, *the one sails* NE b N. 76 *Miles; the other sails* ESE. *to* B, *where she saw the first Ship bear* NW b N.

NWbN. *The Distance of this second Ship from the Port* A, *and from the other Ship at* C, *is demanded?*



In the *Oblique Triangle* ABC, there are Given the Angle CAB=78° 45', or seven Points, (AC being five Points to the North, and AB two Points to the South, of the East Point E). Also the Angle ABC=bAc=33°45'; as being the Difference between NWbN. and WNW. *viz.* three Points; consequently, as the Angles A and B together make ten Points, the third Angle C must be six Points, or 67° 30'. Wherefore all the Angles, and one Side AC=76 Miles, are Given; to find the other two Sides AB and BC.

Whence by *Case* I. of *Oblique Triangles*, there will be found the Distance of the second Ship from the Port A, *viz.* AB=126.4; and from the first Ship at C, *viz* CB=134.1 Miles, as required.

II. *Suppose two Ports* A *and* B *lying* ESE. *and* WNW. *off each other; and from the westermost Port* A, *a Ship sails* NEbN. 76 *Miles; another Ship departs from the Eastermost Port* B, *and sails* 134.1 *Miles, and then meets the former at* C. *Quere the*

the Distance of the two Ports, and the Course the Ship has steered?

In the *Oblique Triangle* ACB, there are Given the Side AC=76 Miles; and the Side BC=134.1; and an Angle opposite to one of them, *viz.* the Angle CAB; for C is five Points to the North, and B two Points to the South of the East Point E; the Angle A then is seven Points or 78° 45′: Whence by *Case* II. of *Oblique Triangles*; there will be found the other Side AB=126.4 the Distance of the two Ports; and the Angle ABC=33° 45′, consequently the second Ship's Course was on the Rumb BC parallel to c A, NW b N. as required.

III. *Admit I sail from a certain Port* A, 126.4 *Miles* ESE; *but by a sudden Change of Wind, am obliged to traverse that Course on the* NW b N. *Rumb* 134.1 *Miles; how far am I from the Port* A, *and on what Point have I made my Way good?*

By the two Given Sides AB=126.4 Miles; and BC=134.1 Miles; and the Angle included ABC= 33° 45′, the Difference of the NW b N. and WNW. Points; there will be found, by *Case* III. of *Oblique Triangles*, the Angle BAC=78° 45′; and therefore, since AB is ESE. the Course from A to C is NE b N; and the Way made good from the Port A to C, will be found AC=76 Miles, as required.

IV. *There are two Ports* A *and* B *lying* ESE. *and* WNW. *off each other, and distant apart* 1264 *Miles; and there is an Island* C *lying Northward, distant from the Westermost Port* A 76 *Miles; and from the Eastermost Port* B 134.1 *Miles, it is demanded on what Points the said Island* C *bears from either Port* A *and* B?

In

In the *Oblique Triangle* ABC, having the three Sides given, the Angles are to be found by *Case* IV. of *Oblique Triangles*; thus the Angle A ſhall be found 78° 45′, and ſhews the Iſland C bears NE b N. from the Port A; and the Angle B will be found 33° 45′, and therefore C muſt bear NW b N. from the Port B, and thus the *Problem* is ſatisfied.

Theſe are all the direct or moſt uſual *Caſes of Oblique Sailing*, which are ſufficient to ſhew how the whole Doctrine of *Oblique angled-Triangles* is concerned to reſolve *Problems* in this Sort of *Navigation*; and alſo how any other *Caſes of Oblique Sailing*, like to theſe here ſpecified, may be eaſily reſolved by any one who has learned what hath been before delivered of *Oblique Trigonometry*.

Thus I have finiſhed a copious Application of the whole Art of *Right-lined Trigonometry* to the various kinds of *Plain* or *Rectilineal Sailing*; what relates to great *Great Circle Sailing*, and to the making and Sailing by the *Globular Chart*, will be delivered in the Second Part of this Work.

Z CHAP.

CHAP. II.

Plain Trigonometry applied to Cosmography, in Measuring the Globe of the Earth, and its several Parts; also of the Atmosphere, Rain-bow, Clouds, *&c.*

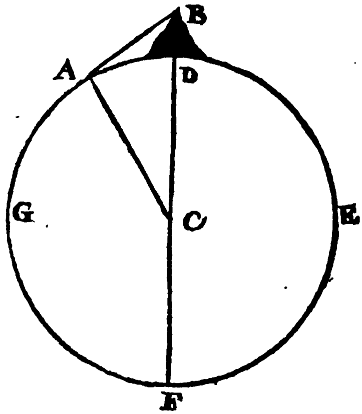
Cosmography, according to the Etymology of the Word, (which is composed of Κοσμ☉, *Mundus, the World*; and γραφὴ, *Scriptum*, a *Description*;) signifies a Description of the World in general, including all that is known or to be known of the Heavens and the Earth. But in a more limited Sense, it is taken for the Science of this terrestial Globe on which we live, with its Appendages, as the *Atmosphere*, its *Meteors*, &c.

I shall apply the Doctrine of *Plain Trigonometry* to *Cosmography* only in this latter Acceptation; and shall shew how by its Service, the Dimensions of the whole Earth, and its several Parts, as the *Torrid, Temperate* and *Frigid Zones*; the Height of the *Clouds*; of the *Atmosphere*; the *Dimensions* of the *Rain-bow*, &c.

And first of all I shall shew how by Trigonometry, the Circumference, Diameter, and consequently the

The Magnitude of the whole Earth was attempted by the Ancients; who for that Purpose used several Methods; as

I. *Suppose, in the adjacent Figure* AEFG *represents a great Circle, or the Circumference of the Earth; and* BD *the perpendicular Height of some Mountain, as* Teneriff *for Instance; and* A *the farthest Point of Sight, from the Top thereof.*



Now by good Instruments, the Angle ABD may be taken, and the Side AB may be known, and the Angle at A being a Right one, there is, in the *Right-angled Triangle* ABC, Given the Side AB=179.5 Miles suppose; and all the Angles supposing B=87° 25′, to find the Side AC or Semidiameter of the Earth.

Then by *Case* II. of *Right-angled Triangles*, say;

As the Angle at the Earth's Center ——	ACB= 2°35′=1.3460893
Is to the Side AB in Miles, *viz.* ———	179.5=2.2541210
So is the Angle at the Summit ———	ABC=87°25′=9.9995636
To the Earth's Semidiameter ———	AC = 3979 =3.5997739

Hence

Hence the Earth's Diameter is about 7958 Mile[s] and 7958 multiplied by 3.1416 will give the Circum[-]ference of the whole Earth about 25020 *Englis[h]* Miles.

II. *Some, when the Mountain is not so very high, proceed by a second Method, thus;*

Suppose BD be the Altitude of a Mountain, equal to ½ a Mile, and the Distance A, the Bound of Vision, be found 44.6 Miles; now the Arch AD being so very small a Portion of the Earth's Circumference, may be taken for a Right-Line; and then in the Triangle ADB, Right-angled at D, there is Given the Side BD=.5, and AD=44.6, *English* Miles; to find the Angle BAD. Now this Angle is found by *Case* IV. of *Right Triangles*, to contain 38 Minutes. But (by Similarity of Triangles) the Angle A is equal to the Angle at C, whose Measure is the small Arch AD; wherefore say, as the small Arch 38′ : is to 44.6 Miles : : so is 60′ : to about 70 Miles; the Miles in one Degree being thus known, the whole Periphery, (which contains 360 Degrees) is easily known.

Or otherwise thus: Because of the similar Triangles ABD and CDA, it will be, as BD : AD : : AD : DC, that is, as ½ a Mile, is to 44.6 Miles; so is 44.6 Miles to 3979 Miles, the Length of the Earth's Semediameter, ; whence the Circumference is found 25020 Miles, as before.

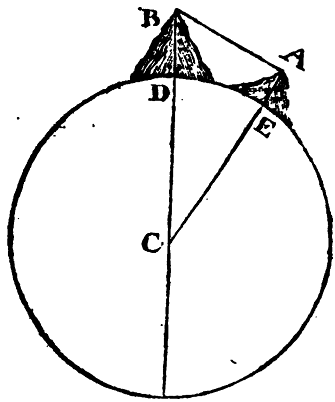
III. *By a* Third Method *the same Things are found thus:*

Suppose that only the Altitude of *Teneriff* BD were Given, 4 Miles; and the Angle ABC to be taken 87°25′ then

...n the Angle at C, is 2° 35′; of this Angle CB is ...Secant, and the Part thereof CD is Radius; hence ...by *a Table of Sines, Tangents, and Secants)* we shall ...ave this *Analogy*, as the Excess of the Secant of ...° 35′ above Radius (*viz.* 1001017—1000000=) 1017, in Parts of the Radius : is to Radius 1000000 : : so is BD=4 Miles, : to DC in Miles, *viz.* 3979, as before

Or thus, as the Excess of Radius above the Sine of the Angle at B : is to the said Sine : : so is BD in Miles : to AC or DC in Miles; when the Height BD is not very great.

IV. *A* Fourth Method *yet more accurate and easy than the foregoing, is thus :*

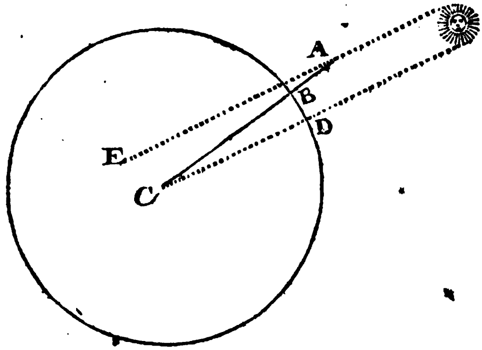


In the adjacent Figure, let B and A be the Tops of two very high Mountains, Towers, &c. and let the Distance between them DE be accurately measured; next with a good Instrument let the Angle CBA, and the Angle CAB be precisely taken on the Top of each Hill; then will the other third Angle at the Center C be known, suppose 1° 25′; and suppose the Measure of that Angle DE, or the Distance of the two Mountains, were found 98 *English* Miles; They make this Proportion, saying;

If the Arch DE=1° 25′ contain 98 Miles, what will the whole Periphery contain, being 360 Degrees? Answer, near 25020 Miles.

V. Erat-

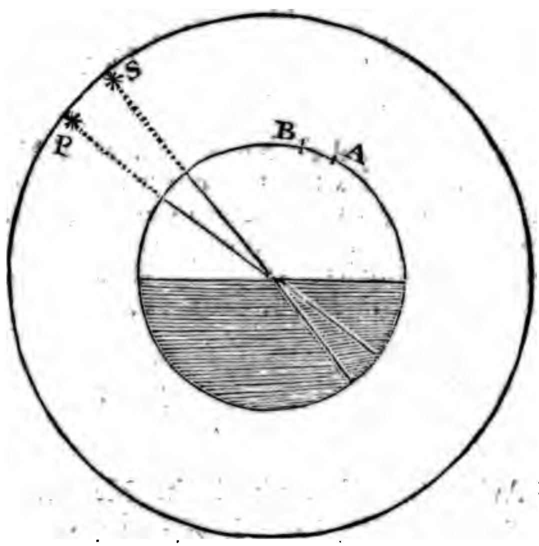
V. Eratosthenes (*who lived about 200 Years befo Christ*) *is reported to have used the following Meth for this Purpose*:



Suppose D be a Place on the Earth on which the Sun's Rays fall perpendicularly and proceeds to the Center C; and B another Place on which at the same Time the Rays fall obliquely; and let the Distance BD be known by Mensuration; when the Observation is made, erect a *Style* or *Staff* AB, on the Spot B, and measure the Angle BAE which is the Compliment of the Sun's Altitude at B, which suppose is found 8° 45′; but because all Rays proceeding from the Sun to the Earth, may be esteemed Parallel (they being of so immense a Length, and the Earth so small a Point in Comparison therewith) therefore the Angle ACD shall be equal to the Angle CAE, by *Theorem* III. But the Arch BD is the Measure thereof, and is known, as being the Distance of the two Places B and D, suppose 610½ Miles; then say, as BCD=8° 45′ : is to the Distance or Arch BD 610.5 Miles, : : so is the whole Periphery 360° Degrees : 25020 Miles nearly. But *Eratosthenes* made the Circumference of the Earth 250000 Furlongs, *i. e.* upwards

rds of 30000 Miles, above 5000 too many; which suppose was owing to the Inexpediency of performing the Experiment with the two Cities of *Cyene* and *Alexandria* in *Egypt*; their Difference of Latitude being too inconsiderable, for such a Purpose. For the Method it self is very exact and conclusive.

VI. *A* Sixth and last Method *I shall mention for finding the Earth's Diameter, Circumference, &c. shall be that which our modern Mathematicians have used as the best and most perfect, and demonstrative of all others.*



In the foregoing Figure; suppose a Person on the Earth's Superficies at A beholds the North Star at P, in a known Elevation; and that then he proceeds directly North from the Point A to B, where, by most exquisite Instruments, he finds the North Pole Star

Star elevated from P to S, just one Degree; consequently if the Distance AB be exactly measured it will give the Number of Miles precisely, in one Degree; and thence the Circumference is most truly known.

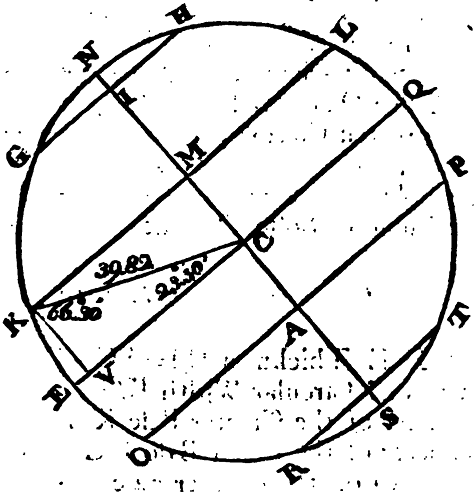
By this Method our Countryman *Norwood* found one Degree to contain 69½ of our *English*; and the two famous *French* Mathematicians, *Picard* first, and *Cassini* afterwards, with the best Advantages of Instruments, Climate, and Place, and by order of their most Christian King, applied the same Method, and found the Measure of one Degree, the same very nearly, as our *Norwood* had done before; he making the Diameter of the Earth 7964 Miles, and they made it to be 7957⅘½ Miles.

These are the most famous Mathematical Methods of measuring the Earth, the four first of which are called Terrestial, the other two Celestial, and they who would see a great deal on this Subject, as concerning its Antiquity, Progress, Deficiencies, &c. may see *Varanius. Lib.* I. *Chap.* IV.

The Mensuration of the Zones of the Earth.

HAving thus determined the Measure of the whole Earth, I shall proceed by Trigonometry to take the Dimensions of those remarkable Portions of the Earth, called the *Zones*; *viz.* The Torrid Zone KLPQ (in the following Figure); the two Temperate Zones KGHL, and OPTR; and the two Frigid Zones GIHN, and RTS. In order to this, there needs only the Solution

tion of the *Right-angled Triangle* CVK, in which there is Given the Side CK= Semidiameter of the Earth, *viz.* 3982 Miles, according to Mr. *Norwood*;



and the Angle KCV=23° 30′ the greatest Declination of the Sun from the Equator EQ, whose Compliment is 66° 30′. Hence by *Case* III. of *Right Triangles*, the Side VC=KM, will be found to be 3651 ½ Miles, and the Side KV=GI=CM will be 1587 ½ Miles.

From hence are inferred the Measures of each Zone as to the Diameter and Circumference thereof (to which, for the Sake of other Purposes, I have added the Square or Superficial, and the Cube or Solid Measures) as follows.

Of the Torrid Zone.

	Miles.
THE Thickness AM=2KV ——	3175.5
The Circular Width KO, or LP, —	326
The greatest Diameter EQ —— ——	7964
The lesser Diameter KL or OP, —— —	7303
The greatest Circumference ——	25020
Square Miles on its Convex Surface —	79452689
Its whole Superficies —— ——	163235401
The Cubic or Solid Content ——	149801827539

Of each Temperate Zone.

THE Thickness IM=VC—CM —	2064
The Circular Width KG ——	2988
Diameter of the Greater Base KL —	7303
Diameter of the Lesser Base HG ——	3172
The Area of the Greater Base ——	41891356
The Area of the Lesser Base ——	7920315
The Circumference of the Greater Base	22943
The Square Miles on its Convex Surface	51636120
The whole Superficies ——	101447792
The Cubic or Solid Content. ——	56005572760

Of each Frigid Zone.

THE Depth NI ——	330
The Diameter GH ——	3175
The whole Width GNH ——	3266
The Circumference of the Base —	9976
The Convex Superficies ——	8262637
The whole Superficies ——	16182952
The Solidity, or Cubic Content —	1326908051

Also,

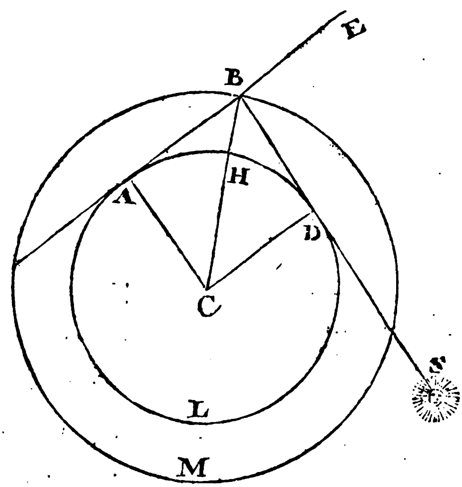
Alſo,

THE Diameter of the whole Earth being 7964
And its Circumference —— 25020
The whole Superficies thereof will be 199250205
And its ſolid Content —— 264466789170

Theſe are the Meaſures of the Terraqueous Globe, and its ſeveral Parts, as found by the Rules of Trigonometry; whereby we ſhall next proceed to find the Height of the Circumambient Atmoſphere.

To meaſure the Atmoſphere.

In order to do this, in the following Figure, let ADL repreſent the Earth's Surface; and BM the Circumference of the higheſt Region of the Atmoſphere, and let BH be the perpendicular Height thereof, which is to be meaſured.



Now for this Purpoſe let S be the Sun depreſſed 18° below the Horizon, in which Caſe, the Morning

and Evening Twilight begins and ends; and let SB be a Ray of Light touching the Earth at D, and paſſing to the upper Region of the Atmoſphere, is there reflected by a Particle of Air at B, in the Line AB, which alſo toucheth the Earth at A.

Now ABE being ſuppoſed the Horizontal Line on the Earth's Superficies, and the Sun being 18° below the Horizon, 'tis plain the Angle SBE is equal to the Sun's Depreſſion, or 18°; and becauſe AB is alſo a Tangent, the Angle ACD is equal to the Angle SBE, the half of which is the Angle ACB, which therefore would be equal to 9°, were it not that the Ray SB is bent by Refraction from BA towards BH about the Quantity of 30′, or half a Degree; and ſo much muſt the Angle ACB be diminiſhed, and ſo its true Meaſure will be 8° 30′.

Hence in the *Right-angled Triangle* ACB, there is given the Angles; and the Side AC, being the Semidiameter of the Earth; to find the Side CB: This by *Caſe* I. of *Right Triangles*, is found thus:

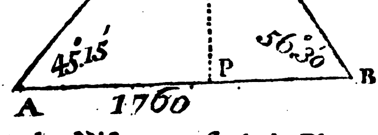
As the Sine of the Angle	ABC=81°30′=	0.0047967
Is to the Semidiameter of the Earth	AC= 3982 =	3.6001013
So is Radius	90°=	10.0000000
To the Side	BC= 4026 =	3.6048980
From which ſubſtract	CH= 3982	
There remains the Height of the Atmoſphere	=	44 *Engliſh* Miles.

In the ſame Figure, and by the ſame Triangle ACB, we may determine another very curious Problem, *viz.* the Proportion of the Sun's Light and Heat when he is in the Spectator's Zenith, and when in

in his Horizon, or when his Rays fall perpendicular as BH, and when in an Horizontal Direction as AB. For the Light and Heat at H will be to that at A in the Reciprocal Proportion of AB to BH. But the Line BH is 44, and by the same Means the Line AB is found 595. Hence AB : BH : : 595 : 44 : : Sun's Light and Heat at H : its Light and Heat at A; that is near as 14 to 1. Hence we see the Reason (says Dr. *Keil*) why without hurting our Eyes we can look upon the Sun at Rising or Setting, but not when he is on the Meridian. And therefore since it is so much weakned in passing through so small a Space as our Atmosphere, if this Atmosphere were so large as to reach to the Moon, and its Density the same, neither the Sun, Moon, or Stars could then be seen.

To measure the Height of the Clouds.

I. *THE common Method for this is to take the Angle the Cloud makes at two different Places, and by two several Persons at one and the same Moment of Time.*

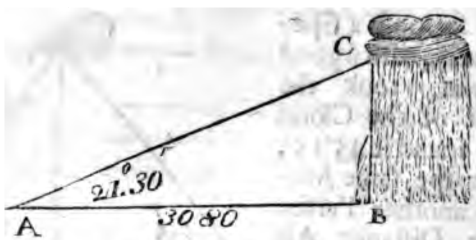


For suppose a Spectator at A, with a good Quadrant, took the Height of the Cloud C, and found it 45°15′, and at the same Moment, another Person at the Distance AB took the Height of the same Cloud, and found it 56° 30′, and let the Distance of their Places of Observation AB be one Mile, or 1760 Yards; then

then in the *Oblique-angled Triangle* ABC, there are known all the Angles, and the Side AB, to find either of the other Sides, ſuppoſe AC; this by *Caſe* I. of *Oblique Triangles*, will be found by the Sliding Rule about 1640; then in the *Right-angled Triangle* APC there is Given the Angle at Baſe A=45° 15′, and the Hypothenuſe AC=1640 Yards nearly, to find the Cathetus CP; this by *Caſe* III. of *Right Triangles*, will be found by Sliding Rule to be about 1450 Yards, or a little above three Quarters of a Mile, and ſo high was the Cloud.

Another Way by one Perſon.

II. *SUppoſe (as I have often obſerved it my ſelf) that* C *be a dark denſe Thunder Cloud, and begins on a ſuddain (with the Thunder Stroke) to rain very hard; and that a Perſon at* A *obſerves the ſtreaming Rain to fall at the known Diſtance* B; *and at the ſame Time takes the Altitude of the Cloud, and finds it* 21° 30′, *and let the Diſtance* AB *be a Mile and three Quarters, or* 3080 *Yards.*

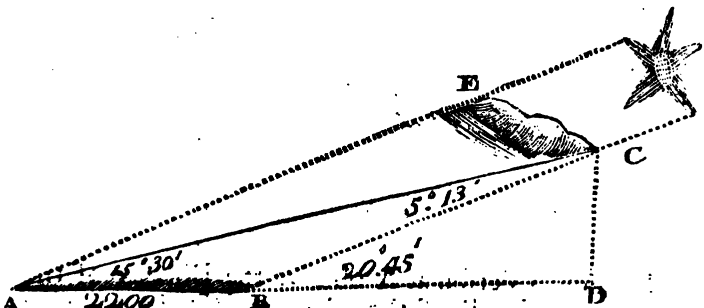


Then in the Triangle ABC, Right-angled at B (for the Rain, in ſtill Weather, falls nearly Perpendicular) there is Given the Diſtance AB, and the Angle at Baſe A; to find the Cathetus CB; which by

by *Case* I. of *Right Triangles*, will be found nearly 1490 Yards, or about $\frac{1}{1}\frac{1}{7}$ of a Mile, which is the Height of the Cloud.

III. *A third Method for Measuring the Height of Clouds, and that by one single Person, may be this:*

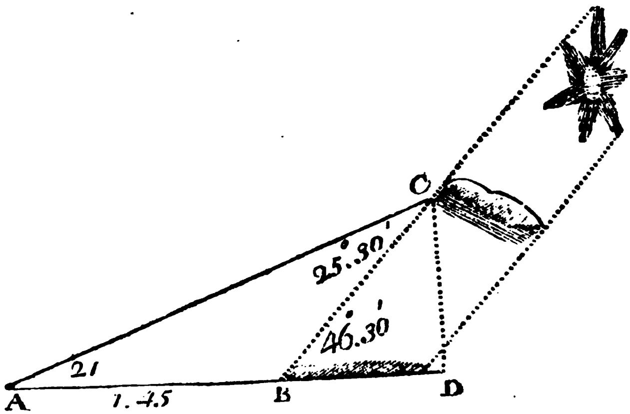
Suppose, as it may sometimes happen, that the Shadow AB of the Cloud C were to touch the Feet of a Spectator at A, at its Extremity; and at the same Time the Person at A could observe nicely the Width of the Shadow AB, or how far it extended from A to B; and that therewith he took the Altitude of the lower Part of the Cloud C.



Now in the Triangle ABC, obtuse angled at B, there is given the Angles A, B, C; for A is known by Observation, suppose 15° 30′; and Angle ABC is the Complement of the Sun's Altitude to 180°. let then the Sun's Altitude be 20° 45′; so shall the Angle ABC be 159° 15′; and the Angle ACB=5° 15′; also the Side AB the Width or Diameter of the Shadow is Given, suppose 1¼ Miles, or 2200 Yards. Then by *Case* I. of *Oblique Triangles*, the Side BC will be found about 6400; and then in the *Right Triangle* BDC, there are known the Angles and the Side

Side BC; whence, by *Case* III. of *Right Triangles*, the Cathetus CD will be found about 2258 Yards, that is, the Height of the Cloud C is somewhat more than a Mile and a Quarter.

IV. *A fourth and last Method I shall mention, is like to the former, but more exact and easy, and is performed likewise by one Person alone, which is thus:*



Suppose a Person at A observe the Distance of the nearest Border B of the Shadow BD, to be 1.45 Mile, at the same Time let him take the Altitude of the highest Part of the Cloud at C, which suppose 21°. And also the Altitude of the Sun, which let be 46° 30′. Then in the *Oblique Triangle* ABC, there are Given all the Angles and the Side AB, whence by *Case* I. of *Oblique Triangles* the Side BC will be found about 1.375; and then in the *Right Triangle* CBD, the Angles and Side BC being Given, there will be found by *Case* III. of *Right Triangles*, the perpendicular Height of the Cloud CD=1.27, a little above $1\frac{1}{4}$ Mile.

These

These are the principal Methods of taking the Altitudes of Clouds, I could have suggested more; but the three last of these, each of which may be performed by a single Person, are new to me, having never seen them published by any Writer, nor are they mentioned even by *Varenius* himself, the most exact, accute, and laborious Author that ever wrote of these Affairs.

To Measure the Rainbow.

IN order to take the Dimensions of this most wonderful Phænomenon, the most grand and beautiful Piece of Nature's Gallantry, I shall premise a few Things relating to its Formation, from that great Father of modern Philosophy, Sir *Isaac Newton*.

I. First it must be observed, that the Bow is not really in the Cloud which rains, but in the Rain it self, or falling Drops of Water, and therefore must always be seen below its proper Cloud.

II. The Bow must appear diametrically opposite to the Sun; for a Ray must pass from the Sun at S, through the Eye of the Spectator at O, to the Center of the Rainbow P. See the following Figure.

III. The Colours of Light consist promiscuously in the Sun's Rays, and are to be exhibited distinct and singly by Reflection and Refraction; and those Rays are reflected and refracted most which are of the deepest blue Colour, and those least which are of the brightest Red.

IV. The interior Bow, as AEFB, is formed in round Drops of Rain by two Refractions of the Sun's Rays, and one Reflection between them, as in the Drops E, F.

V. The lower Part of this Bow consists of those Drops, in which the most refrangible Rays can after one Reflection be refracted to the Eye; the Angle of this Refraction is POE (=OES) 40 Degrees and 2 Minutes; hence all the Drops in the Line OE shall strike the Eye with the most refrangible Ray of the deepest Violet Colour in that Region which it describes.

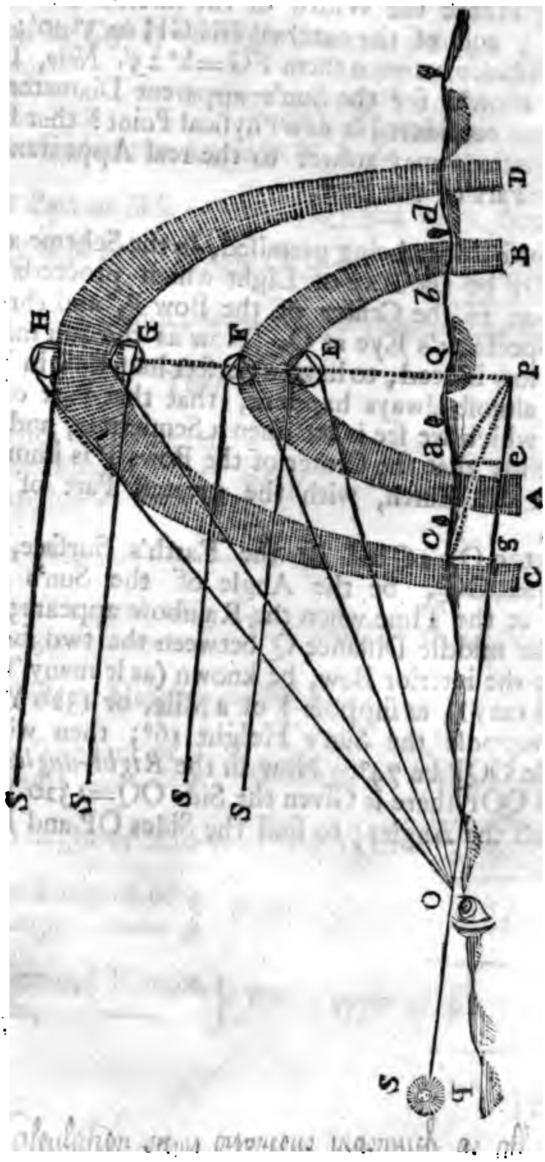
VI. The highest Part of this interior Bow contain those Drops in which the least refrangible Rays can after one Reflection be refracted to the Eye; the greatest Angle of this Refraction is POF (=OFS) =42 Degrees, 17 Minutes. Hence all the Drops in the Line OF shall tinge the upper Part of the Bow, which it describes, with the deepest red Colour.

VII. The exterior Bow CGHD is made in Drops of Rain, by two Refractions, and also by two Reflections, as in the Drops G, H.

VIII. The lower Part of this Bow consists of those Drops, in which the least refrangible Rays can after two Reflections, be refracted to the Eye; and the least Angle of this Refraction, is POG (=SGO) $=50^\circ 42'$.

IX. The highest Border of this Bow is made by Drops which refract the most refrangible Rays possible after two Reflections: and the least Angle thereof is POH (= SHO) $=54^\circ 22'$; hence the Colours of this Bow will be in an inverse Order, to those of the other Bow.

X. Hence


H
G
F
E
D
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C
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X. Hence the Width of the internal Iris EF is $2^\circ 15'$; and of the external Iris GH $= 3^\circ 40'$; and the Distance between them $FG = 8^\circ 25'$. *Note*, I have here allow'd for the Sun's apparent Diameter $30'$, and not considered it as a Physical Point? that so the Calculations may answer to the real Appearance of every Part of the Bow.

These things being premised, in the Scheme above, let SOP be the Ray of Light which proceeds from the Sun to the Center of the Bow P, and through the Spectator's Eye at O. Now as the Sun must be very low indeed, to have a perfect half Bow in View, so it almost always happens, that that Part of the Bow which we see is less then a Semicircle, and consequently that the Center of the Bows P is immersed under the Earth, with the greatest Part of each Bow.

Let qOcaQbd be the Earth's Surface, and $SOq = POQ$, be the Angle of the Sun's Altitude at the Time when the Rainbow appears; and let the middle Distance Q between the two Legs a, b, of the interior Bow, be known (as it many Times easily may); as suppose $\frac{3}{4}$ of a Mile, or 1320 Yards; and suppose the Sun's Height 16°; then will the Angle OQP be 74°. Now in the *Right-angled Triangle* QOP there is Given the Side $OQ = 1320$ Yards, and all the Angles; to find the Sides OP and PQ,

Then

Then by *Case* I, of *Right Triangles*; say,

As Radius	———	90°	=10.
Is to the Side	——	OQ=1320=	3.1205739
So is the Sine of the Angle		POQ=16°	= 9.4403381
To the Part or Side	———	PQ=364	= 2.5609120
So is the Sine of the Angle		OQP=74°	= 9.9828416
To the Base or Side,	———	OP=1269=	3.1033155

Then say,

As Radius is to the Base—	OP= 1269 =	3.1033155
So is the Tangent of the Angle ———	POE=40° 2′=	9.9549455
To the Side ———	PE= 1143 =	3.0582610
So is the Tangent of the Angle ———	POF=42°17′=	9.9587542
To the Side or Height ——	PF=1153½=	3.0620697
So is the Tangent of the Angle ———	POG=50° 42′=	10.0869863
To the Height or Side ——	PG= 1550 =	3.1903018
So is the Tangent of the Angle ——	POH=54°22′=	10.1445959
To the greatest Height, or Side, ——	PH= 1770 =	3.2479114

Then fay again,

As Radius is to the Bafe	OP=	1269 =	3.1033155
So is the Secant of the Angle	POE=	40° 2′=	10.1159582
To the Side or Hypothenufe	OE=	1656 =	3.2192737
So is the Secant of the Angle	POG=	50° 42′=	10.1983351
To the Side	OG=	2003 =	3.3016506

In the next Place, fay;

As Radius, to the Side	P a=PE=	1143 =	3.0582610
So is the Sine of the Angle	a PQ=Pa e=	71° 27′=	9.9768296
To the Side	a Q=	1084 =	3.0350906

Laftly fay,

As Radius to the Side	P c=PG=	1550 =	3.1903081
So is the Sine of the Angle	c PQ=Pc g=	76° 26′=	9.9877099
To the Side	c Q=	1507 =	3.1780117

Now to find the Circumferences of the middle Parts of each Bow, proceed thus;

Multiply — 2PE+EF=	2296.5
By	3.1416
The Product is	7214.6844

Again,

Again,

Multiply	2PG+GH=	3320
By		3.1416
The Product is		10430.112

From this general Calculation of the Rainbow aforegoing, we find the Measures of its several Parts, and their Distances from the Spectator's Eye to be as follows:

1st. The least Semidiameter PE, of the interior Bow, is 1143 Yards, or about 0.64 of a Mile; whence the Diameter is 1.28 Mile.

2d. The greater Semidiameter PF=1153½ Yards =0.65 of a Mile, whence the greatest Diameter is 1.3 Mile.

3d. The Breadth thereof EF is 10½ Yards, or 31½ Feet.

4th. The Circumference of the Middle thereof is 7214.68 Yards, or 4.09 Miles.

5th. The least Semidiameter of the exterior Iris, PG=1550 Yards, or 0.87 of a Mile; Whence the least Diameter is 1.74 Mile.

6th. The greatest Semidiameter thereof PH=1770 Yards, or 1.5 Mile, whence the greatest Diameter is about 3 Mile.

7th.

7th. The Breadth thereof GH is 220 Yards, or 660 Feet.

8th. The middle Circumference thereof is 10430 Yards, or about 5.92 Miles.

9th. The Diſtance between each Bow FG=396½ Yards.

10th. The Heighth of the interior Bow above the Horizon is QE=779 Yards, or 0.45 Mile.

11th. The Heighth of the exterior Bow QG=1186 Yards, or 0.66 Mile.

12th. The Center of the Bows P is ſubmerſed by the Surface of the Earth the Length of PQ=364 Yards.

13th. The Center P is diſtant from the Spectator's Eye, by PO=1269 Yards, or ¾ of a Mile.

14th. The Diſtance of the interior Bow OE=Oa =1656 Yards, or $\frac{9}{10}$ of a Mile.

15th. The Diſtance of the exterior Bow OG=Oc =2003 Yards, or 1.13 Mile.

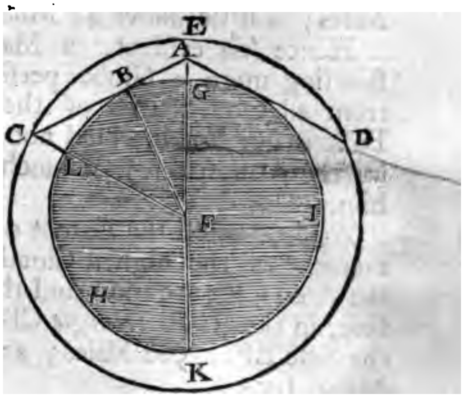
16th. The Diſtance of the Legs of the interior Bow on the Earth's Surface, is ab=2168 Yards; or 1.2 Mile, or a little more.

17th. The Diſtance of the Legs of the exterior Bow cd=3014 Yards, or 1.7 Mile.

And

And ſuch are the Dimenſions of a Rainbow at the given Diſtance and Altitude of the Sun aforegoing; which as it is ſo great a Piece of Curioſity, muſt needs be acceptable to every Perſon of Ingenuity and Taſte.

To determine the fartheſt Bounds of Sight; or the viſible Horizon.



LET BHI repreſent the Globe of the Earth; CEDK, the Circumference of the Cloudy Heavens. Alſo let AG be the Height of a Man of 6 Feet in Stature; then 'tis evident (AC being drawn a Tangent to the Surface of the Earth in B) that the Point B is the fartheſt Bound of Viſion on the Earth's Superficies; and C in the cloudy Expanſe; ſuppoſe the Height thereof CL, or EG, to be 1 Mile.

Now becauſe the Semidiameter of the Earth BF, is found to be 3982 Statute Miles, and in every ſuch Mile there is 1760 Yards; therefore 3982x1760= 7008320, the Yards in a Semidiameter of the Earth.

Hence there being known, in the *Right Triangle* ABF, the Side BF=7008320 Yards, and the Side AF=7008322 Yards; we can eaſily find the Angle AFB, by *Caſe* V, thus;

As the Side —	AF=7008322=	3.8456143
Is to the Radius —		10.0000000
So is the Side —	BF=7008320=	3.8456140
To the Sine of the Angle BAF=	89° 56′ =	9.9999997

Whence the Complement thereof BFA, or the Arch BG, will be 4 Minutes; which is thus converted into Miles; as 60° : 69½ Miles :: 4′ : 4.63 Miles; a little above 4½ Miles.

Hence 'tis evident, a Man of 6 Feet Stature, ſtanding upon a Globe perfectly round, and free from all Obſtructions of the Sight, of the ſame Bulk as our Earth, could not by his Sight (unaſſiſted by the Atmoſphere) ſee much above 4½ Miles about him.

Again, to find the Extent of the Sight in the Firmament of the higheſt Clouds, *viz.* AC, or rather the Arch CE; we muſt find the Angle BFC; therefore, in the *Right Triangle* CBF, there being known the Side BF=3982 Miles; and the Side CF=3983 Miles, ſay;

As the Side —	CF= 3983 =	3.6002103
Is to Radius —	—	10.0000000
So is the Side —	BF= 3982 =	3.6001013
To the Sine of the Angle	BCF=88°43′=	9.9998910

Hence the Complement thereof BFC is 1° 17′, to which if we add the Angle BFA=4′, we ſhall have the Angle AFC=1°21′; whoſe Meaſure on the Earth, is the Arch LG, and in the Clouds, CE. The

Quantity

Quantity of the Arch LG=81′ is thus found; as 60′ : 69½ Miles : : 81′ : 93.82 Miles. But as FG =3982 : FE=3983 : : LG=93.82 : CE=93.84 Miles, the Extent every Way of the visible Horizon terminated by the Clouds.

Whence it follows, that the greatest Extent of the visible cloudy Expansion is the Arch CD=2CE= 187.68 Miles; and the Circumference of the sensible or visible Horizon terminated in the Clouds, is very nearly three Times 187.68=563 Miles. However a Method for calculating it to the greatest Nicety is hereby made plain and evident, and shall (with other Things of this Nature) be left to the Learner's Exercise.

CHAP. III.

Plain Trigonometry applied to Astronomy; in the Mensuration of the Distances, Magnitudes, Places, and other Affections of the Heavenly Bodies, which compose our Solar System.

ASTRONOMY, the darling Science of all the Sons of Art; the most noble and splendid Branch of the Encyclopædia, or whole Compass of Literature; the very Zodiack of the Celestial Sphere of Knowledge; I say this excellent Art depends altogether

on Trigonometry, plain and ſpherical; the common Aſtronomical Problems, ſo neceſſary to Sailors and Almanack-Makers, are performed by ſpherical Triangles, of which I may treat enough of in that Part of this Work; but the Fundamentals of the Art, the grand Problems of the Planatary Diſtances, of the Penumbra and Shadow of the Earth and Moon, of Eclipſes, of the Anomalies of the Planatary Motions, and the Explications of the Theories of both Planets and Comets, all depend upon and reſult from the Calculations made, for the greateſt Part, by the Rules of Plain Trigonometry, as appears evident by what here follows concerning thoſe Matters.

But I think it neceſſary to give the young Tyro a View of the Solar Syſtem, and an Account of its ſeveral Conſtituent Parts, before we proceed to meaſure them; which therefore is as follows.

I. *A Deſcription of the Solar Syſtem.*

I. IN the Scheme of this Syſtem of the World, the Sun occupies the middle Point or Center; and has no other Motion, but round its own Axis once in about 25 Days; this Motion is diſcoverable by the Macula or dark Spots on his Disk.

II. Next to the Sun is the Orb of *Mercury*, in which this Planet moves about the Sun once in 87 Days, and 23 Hours.

III. The next Planet which revolves about the Sun is *Venus*, that bright Planet which we call the Morning and Evening Star; ſhe performs one Revolution in about 224 Days and 17 Hours.

IV.

IV. The third Planet from the Sun, is our Earth with its single Satellite; the Earth performing one Revolution round the Sun in 365 Days, 5 Hours, and about 50 Minutes; and the Moon round the Earth once in 27 Days, 7 Hours, and 43 Minutes.

V. *Mars* is the next Planet, which circulates about the Sun once in 686 Days, 23 Hours, and 27 Minutes; and round his own Axis once in 24 Hours, and 40 Minutes.

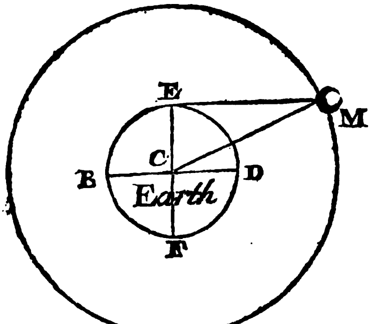
VI. The next superior Planet is *Jupiter*, with his 4 Satellites; he revolves about the Sun once in 11 Years, 317 Days, 12 Hours, and 20 Minutes; and about its own Axis, once in 9 Hours, and 56 Minutes. The Periodical Revolutions of his Satellites or Moons about his Body are as follows; of the 1st, in 1 Day, 18 Hours, 27 Minutes. Of the 2d, in 3 Days, 13 Hours, 13 Minutes. Of the 3d, in 7 Days, 3 Hours, 42 Minutes Of the 4th, in 16 Days, 16 Hours, 32 Minutes.

VII. The highest, and vastly the farthest Distant, of all the Planets, is *Saturn*, who wanders in perpetual Solitude through the Desert Extremities of the Mundane Space, attended by his 5 Satellites or Moons, and finishes his slow Revolution about the Sun, once in 29 Years, 174 Days, 6 Hours, and 36 Minutes; and about his own Body revolve his 5 Moons the following Periods of Time; The first in 1 Day, 21 Hours, 18 Minutes; The second in 2 Days, 17 Hours, 41 Minutes; The third in 4 Days, 12 Hours, 25 Minutes; The fourth in 15 Days, 22 Hours, 41 Minutes; The fifth in 79 Days, 7 Hours, 48 Minutes.

VIII. Beyond the vaſt Diſtricks of the Planatary Syſtem, are placed at an amazingly immenſe Diſtance, the fixed Stars; and by that Means are they without the Reach of the largeſt and moſt accurate Mathematical Inſtruments, and the Cognizance of the moſt ſegacious and penetrating Aſtronomers and Philoſophers.

II. *To meaſure the Diſtance of the Moon from the Earth.*

HAving thus Given a Curſory View of the moſt antient as well as modern and true Syſtem of the lower World, I now proceed to the Dimenſions of the ſeveral Parts thereof; and firſt of the Diſtance of the Moon from our Earth, as follows.



In the Figure adjoined let BEDF repreſent the Earth, and M the Moon in its Orb, ſeen in the Horizon; then the Angle EMC will be the apparent Semidiameter of the Earth EC ſeen at the Moon M; which Angle is called the Parallax; and when the Moon is ſeen in the Horizon, 'tis the Horizontal Parallax of the Moon, and is nothing but the Difference of her true and apparent Diſtance from the Zenith. This Angle is greateſt when the Moon is in her Perigœum, and her Orb moſt excentric; being then according to Mr. *Whiſton's* Tables, $1^{\circ}\ 2'\ 10''$. And this I ſhall chuſe for Calculation; becauſe in ſuch a Caſe the

Moon

Moon is the leaſt Diſtant from the Earth it can poſſibly be.

Therefore in the *Right-angled Triangle* CFM, there is given all the Angles, and the Side EC, being the Semidiameter of the Earth, or 3982 Miles, to find the Side CM; by *Caſe* I, of *Right Triangles*, ſay thus;

As the Sine of the Moon's Horizontal Parallax.	M=1° 2′ 10″=	8.2574189
Is to one Semidiameter of the Earth ——	EC = 1 =	0.0000000
So is the Co-Sine of the Parallax ————	C=88° 57′ 50″=	9.9999290
To the Side (in Semidiameters of the Earth)	EM = 55.27 =	1.7425101
So likewiſe is Radius ————	90° =	10.0000000
To the Side (in Semidiameters of the Earth)	CM = 55.29 =	1.7425811

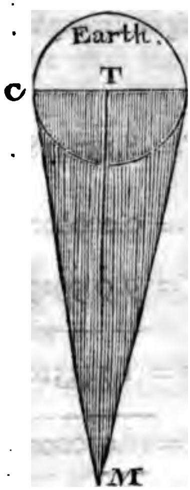
Hence then 55.27×3982=220085.1 *Engliſh* Miles, the Diſtance of the Moon from the Spectator.

Alſo 55.29×3982=220344,78 Miles, the Diſtance of the Moon from the Center of the Earth.

'Tis ſeldom the Moon come ſo very near the Earth, her Diſtance is variouſly determined by various Authors from 52 to 64 Semidiameters of the Earth, including the leaſt and greateſt Diſtance.

III. To

III. *To measure the Height of the Earth's Shadow.*



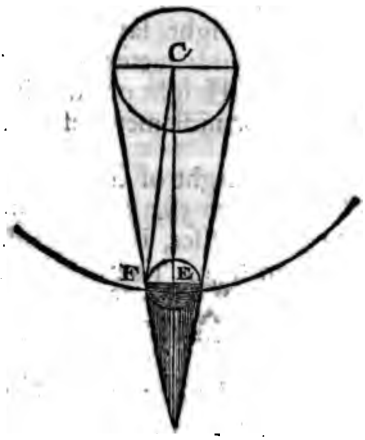
IN the adjoined Figure let CT be the Earth's Semidiameter, TM the Height of the Earth's conical Shadow, and the Angle TMC, being half the Angle of the Cone, is equal to the apparent Semidiameter of the Sun, which in its mean Distance, is about 16 Minutes; therefore in the *Right-angled Triangle* CTM, there are given all the Angles and the Side CT, to find the Cathetus TM, which is the Height of the Shadow; Wherefore by *Case* I, of *Right Triangles*, say;

As the Tangent of —— TMC=	16′	=	7.6678492
Is to one Semidiameter —— CT=	1	=	0.0000000
So is Radius ————————			10.0000000
To the Height ———— TM=	214,8	=	2.3321508

Therefore TM is 214.8 Semidiameters of the Earth, which multiplied by 3982, gives 855333.6 *English* Miles for the Height of the Earth's Shadow. But when the Sun is at the greatest Distance, the Angle TMC is but about 15′ 50″, and then the Height of the Shadow will become 217 Semidiameters of the Earth.

IV. *To*

VI. *To Meaſure the Diameter of the Moon, and Height of her Shadow.*



IN the Scheme adjoined, let C be the Center of the Earth, and E the Center of the Moon; then is ECF = the apparent Semidiameter of the Moon ſeen from the Earth, which is (for the beforementioned Horizontal Parallax) 16′ 52″. The Line CE is the Diſtance of the Moon from the Earth as before found, 55.29 Semidiameters; In the *Right Triangle* FEC there is given then the Angles, and the Side CE; to find the Side FE or Semidiameter of the Moon, ſay thus;

As the Co-Sine of the apparent Semidiam.	EFC=89°43′08″	=9.9999947
To the Moon's Diſtance from the Earth	CE=55.29	=1.7425101
So is the Sine of the apparent Semidiamet.	FCE= 16′52″	= 7.6906627
To the Moon's Semidiameter, in Semidiameters of the Earth	FE=.2711	=.9.4331781

Now 3982x,2711=1079.52 *Engliſh* Miles, for the Moon's Semidiameter, wherefore the Moon's Diameter is 2159.04 Miles.

And becauſe Cones are ſimilar Bodies, their Heights will be in Proportion to the Baſes; But we have the Baſe and Height of the Earth's Cone of Shadow, and the Baſe of the Moon's, wherefore to find its Height, ſay;

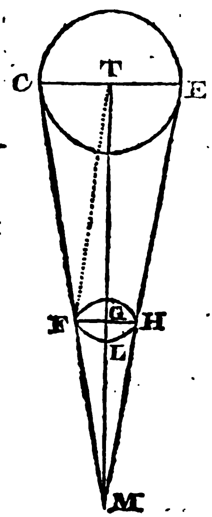
As the Semidiameter of the Earth —— 1 = 0.0000000
Is to the Height of its Shadow 214.8 = 2.3321588
So is the Semidiameter of the Moon ,2711 = .9 4331781

To the Height of its Shadow —— 58.25 = 1.7653369

Therefore 58.25 Semidiameters of the Earth, or 231951.5 Miles, is the Height of the Moon's Shadow.

From hence 'tis evident the Moon muſt be, at the Time of a Solar Eclipſe, within 58 Semidiameters of the Earth diſtant from the Earth's Superficies.

V. *To Meaſure the Diameter of the Earth's Shadow at the Diſtance of the Moon.*



IN the adjoined Figure, let T be the Center of the Earth; CMT half the Angle of the Cone, 16′; FLH the Section of the Shadow at the Diſtance of the Moon TG, whoſe Diameter FH is ſought; In order to find which, we have given, in the *Right-angled Triangle* GMF, the Side GM = (TM—TG, or 214.8—55.29=) 159.51 Semidiameters of the Earth, and all the Angles; therefore ſay;

As

As Radius ———————— 90° = 10.0000000
Is to the Side ———————— GM = 159.51 = 2.2027610
So is the Tangent of GMF = 16′ = 7.6678492

To the Side ———————— FG = ,7423 = .9.8706102

Whence 2 FG = FH = 1.4846. But 1.4846 x 3982 = 5911.677 Miles; and so wide is the Earth's Shadow at the Moon, when she is nearest the Earth; That is, above twice the Moon's Diameter; in which Case Lunar Eclipses, are of the greatest Duration possible.

VI. *To find the apparent Semidiameter of the Earth's Shadow at the Moon.*

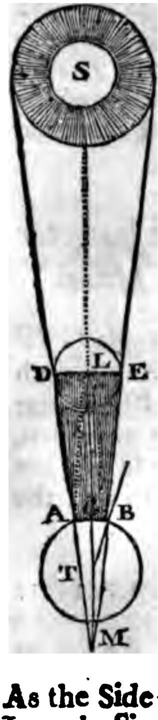
THIS is no more than to find the Angle GTF in the last Figure, for under that Angle doth the Semidiameter of the Earth's Shadow FG appear at the Distance of the Moon; in order to which, there is given in the *Right-angled Triangle* GTF, the two Sides GF = .7423, and GT = 55.29 to find the Angle at T, therefore say thus;

As the Side ———————— GT = 55.29 = 1.7425811
Is to Radius ———————— 90° = 10.0000000
So is the Side ———————— FG = .7423 = .9.8706102

To the Sine of the Angle GTF = 46′10″ = 8.1280291

And such is the apparent Semidiameter of the Earth's Shadow in that Case.

VII. *To measure the total Shadow of the Moon at the Earth, in the Time of a Solar Eclipse.*



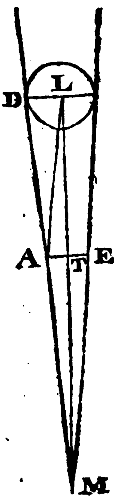
FIRST for the Measure of the Moon's total, or dark Shadow at the Earth in the Time of an Eclipse of the Sun; In this Figure let S be the Sun, L the Moon, and T the Earth; Now in order to find the Width AB of the Moon's Shadow on the Earth, we have Given, in *Oblique Triangles* TMB, the Side TB=1 Semidiameter of the Earth, and TM=3 Semidiameters (for LM =58.25, and LT=55.29 as found, but TM=LM−LT=3, very nearly). Also the Angle TMB=16′, the Sun's Semidiameter; to find the Angle TBM. This is done by *Case* II, of *Oblique Triangles*, thus,

As the Side————————	TB=1=0.0000000
Is to the Sine of——————	TMB=16′=7.6678445
So is the Side———————	TM=3 =0.4771212
To the Sine of the Angle	TBM=49′7″=8.1449657

But

But the Angle CTB = TMB+TBM=65′ 7″, (by *Theorem* V.) and ſuch is the Arch CB, the Double of which is AB=2° 10′ 14″, or 150$\frac{1}{3}$ Miles. *Note*, this Calculation anſwers to a Central Eclipſe, at a middle Diſtance of the Sun; but were the Sun at the Greateſt from the Moon, her Shadow would be longer, *viz.* near 60 Diameters of the Earth, and then the Breadth of the Shadow AB on the Earth's Surface would be about 180 Miles.

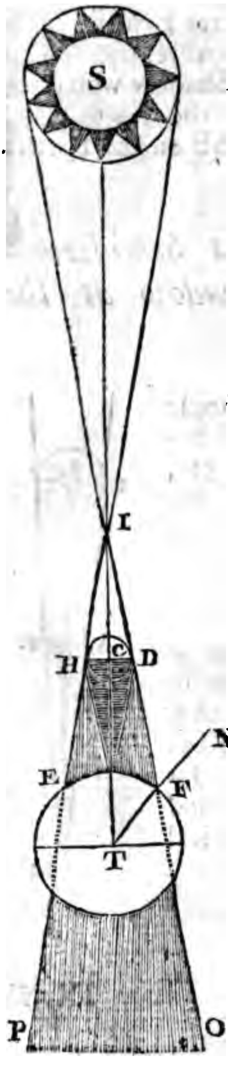
VIII. *To find the apparent Semidiameter of the Moon's Shadow at the Earth.*



IN the adjoining Figure, the Angle AMT=16′ the Sun's apparent Semidiameter, the Angle DAL=16′ 52″, the apparent Semidiameter DL of the Moon ſeen from the Earth T. But this Angle DAL = AMT+ALT; conſequently DAL−AMT=ALT the Angle under which the Semidiameter of the Moon's Shadow at the Earth is ſeen at the Moon, which therefore is equal to 52″; and thus the whole Diameter AE of the Shadow, is 1′ 44″. *Note*, the Semidiameters of the Sun and Moon, for any Diſtance in their Orbs, are found ready calculated in the Aſtronomical Tables.

IX. *To*

IX. *To measure the Penumbra or partial Shadow of the Moon.*



THE Nature and Production of the *Penumbra*, or that large diverging partial Shadow, in form of a Cone, involving the Moon and its dark Shadow, the Reader will find fully treated in Books of Astronomy; I shall here only present him with the Figure and Dimension of it as follow.

Let S be the Sun, C the Moon, and T the Earth; Then is the Figure PIO the Penumbra, or fainter Shadow of the Moon, the Part of which HID above the Moon, is of equal Height, and therefore of equal Magnitude, with the Moon's dark Shadow; all this demonstrated by Astronomers.

Hence then TI (being equal to the Distance of the Moon TC=55.29, and the Length of her Shadow (or upper Part of the Penumbra) CI=58.25) is equal to 113.54 Semidiameters of the Earth; and the Semiangle of the penumbral Cone TIF=16′, or the Sun's Semidiameter. Also the Side TF =1 Semidiameter of the Earth.

Hence

Hence in the *Oblique Triangle* TIF, there is Given two Sides TI and TE, and an Angle opposite to one of them TIF; to find the Angle IFT, or its Complement IFN; by *Case* II, of *Oblique Triangles*, say thus;

As the Side ————————	TF=1	=0.0000000
Is to the Sine of—— ——	TIF=16″	=7.6678445
So is the Side ——— ——	TI=113.54	=2.0551488
To the Sine of—— ——	IFN=31° 54′	=9.7229933

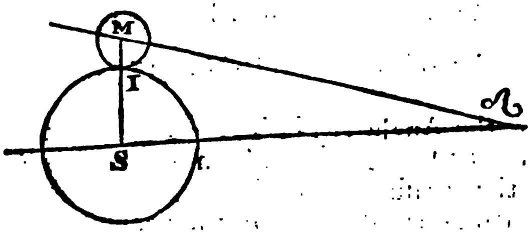
Now the Angle IFN=TIF+ITF; hence IFN—TIF=ITF=31° 38′, which doubled is 63° 16′, and is the Measure of the Arch EF, or so much of the Earth as is obscured by the penumbral Shadow of the Moon, in an Eclipse; that is about 4397 Miles; and when the Earth is nearest the Sun, and the Moon farthest from the Earth, this Penumbra will extend about 4900 Miles, on the Earth's Superficies.

Note; After the same Manner as the Semidiameter of the Moon's dark Shadow at the Earth seen from the Moon, was proved equal to the Difference of the Semidiameters of the Sun and Moon seen from the Earth, so the Semidiameter of the Penumbra at the Distance of the Earth is proved equal to the Sum of the said Semidiameters of the Sun and Moon, *viz.* 32′ 52″; whence the whole Diameter of the Penumbra at the Earth seen from the Moon will be 1° 5′ 44″.

X. To

X. *To determine the Limits of an Eclipse of the Moon.*

LET ☊S represent a Portion a Line in the Plane of the Ecliptic Parallel to the Earth's Orbit;



☊M a Portion of the Moon's Orb; M the Center of the Moon; S the Center of the Earth's Shadow at the Distance of the Moon; ☊ the Node; and MS the Latitude of the Moon when the Shadow of the Earth doth just touch the Disk of the Moon; which Latitude is equal to the Sum of the Semidiameters of the Moon and Shadow, MI and IS; also the Angle M☊S is the Inclination of the Moon's Orb to the Plane of the Ecliptic. Now when this Angle is biggest, and the Semidiaters of the Moon and Shadow are least of all, then is the Point S the nearest possible to the Node ☊ when an Eclipse can happen. Therefore the Distance S☊ is the Limit of Lunar Eclipses in such a Case. But when the said Angle is least, and the Semidiameters greatest, then is S☊ the greatest Limit, beyond which no Eclipse can happen.

Now in Order to find the least Limit, the Angle S☊M must be made 5° 18′, and the Sum of the Semidiameters

midiameters, when least of all, is about 53′=SM; there being given in the Right-angled Triangle ☊SM, the Side SM, and the Angle ☊, to find the Side S☊; say thus, by *Case* II. of *Right-angled Triangles*;

As the Sine of ——	S☊M	= 5°18′	= 8.9655337
To the Side ————	SM	= 53	= 1.7242759
So is the Sine of the An.	SM☊	= 84°42′	= 9.9981393
To the Side (in Min.)	S☊	= 571.4	= 2.7568815

Hence the Least Limit of a Lunar Eclipse is about 9° 31′ 24″, within which there must be an Eclipse.

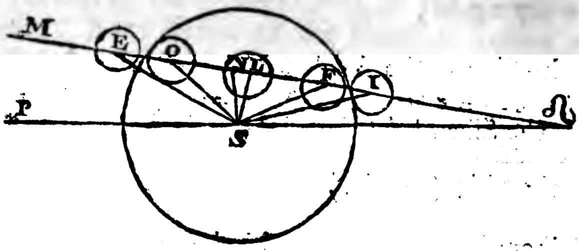
To find the Greatest Limit, the Angle ☊ must be taken 5°, and the Greatest Sum of the Semidiameters, or Line SM, is about 63.2 Minutes. Therefore say again;

As the Sine of the Angle	S☊M	= 5°00′	= 8.9402960
Is to the Side ———	SM	= 63.2	= 1.8007171
So is the Sine of the Ang.	SM☊	= 85°00′	= 9.9983442
To the Side (in Min.)	S☊	= 722.4	= 2.8587653

Therefore the Greatest Limits are 12° 2′ 24″ on each Side the Node, beyond which there can be no Eclipse.

XI. *To calculate the Angles of Incidence and Exit, of Total Immersion and Emersion, the Motion of Half Duration, both of the total Eclipse and of absolute Darkness; and how to turn the Whole into Time, in a Lunar Eclipse.*

THE following Figure is a Representation of a total Eclipse of the Moon; in which P☊ is a Portion of a Line or Path of the Earth's Shadow at the Moon parallel to the Ecliptic; M☊ the Way of the Moon's horary Motion from the Sun, at the



Time of the true Opposition; S the Center of the Earth's Shadow at the Moon; I the Center of the Moon the Moment she enters the Earth's Shadow; F the Moon totally immersed in the Shadow; SL the Least Distance of the Centers of the Earth's Shadow and Moon; NS the Moon's Latitude from the Ecliptic; O the Moon when she begins to emerge out of the Shadow; and E the Moon in her Exit out of the Shadow entirely.

By

By Computations of another Kind, and in the Astronomical Tables, there are given, in any Eclipse, the Semidiameters of the Shadow and the Moon; the Angle M ☊ P, or the Angular Motion of the Moon from the Sun; S N the Moon's Latitude; and S L the Least Distance of the Centers; whence the Rest may be found by the following *Trigonometrical Analogies*;

I. *For the Angles of Incidence and Exit*, I S L,

As the Sum of the Semidiameters ——— S I,
Is to the Radius;
So is the Least Distance of the Centers ——— S L,
To the Co-sine of the Angle of Incidence LIS.
But the Angle of Incidence ISL = ESL the Angle of Exit,

II. *For the Angle of total Immersion and Emersion* FSL.

As the Difference of the Semidiameters ———S F,
Is to the Radius;
So is the Least Distance of the Centers ——— S L,
To the Co-sine of the Angle of Immersion SFL,
Whence FSL=OSL are both known.

III. *For the Motion of Half Duration* I L *of the Eclipse.*

As the Radius,
Is to the Sine of the Angle of Incidence ISL;
So is the Sum of the Semidiameters ——— S L,
To the Motion of Half Duration of the Eclipse I L.

IV. *For Half Continuance of absolute Darkness* F L.

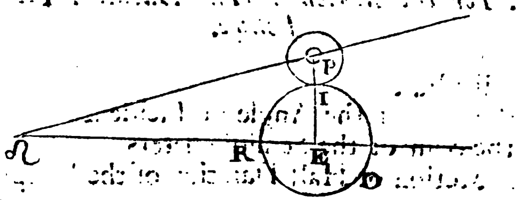
As Radius to the Sine of the Angle of Immersion } FSL;
So is the Difference of the Semidiameters — F S,
To the Motion of Half Continuance of total Darkness } F L.

Thus the Motions are found, the Times in which they are performed (the Horary Motion of the Moon being always given in the Tables) are found by the Golden Rule thus;

As the Horary Motion of the Moon from the Sun in an Hour }
Is to the Time of one Hour, or 60 Minutes;
So is the Motion of Half Duration of the Eclipse,
To the Time in Hours or Minutes of that Duration.
And, So is the Motion of Half Continuance of absolute Darkness }
To the Hours or Minutes of that Half Darkness.

Thus is the Beginning, Middle, and End of a Lunar Eclipse to be found in Motion and Time.

XII. *To determine the Limits of an Eclipse of the Sun.*



In this Figure, Let R O I represent the Disk of the Earth; ☊E a Portion of the Ecliptic; ☊P a

a Part of the Moon's Way from the Sun; E☊P the Angle of the Moon's Way from the Sun, which when least, is 5° 30′, and when greatest, is about 5° 46′. P the Penumbra, in the Middle of which is a small Circle representing the dark Shadow of the Moon; lastly, EP is the nearest Distance of the Disk and Penumbra (being the Sum of the Semidiameters of each EI and IP,) which, when least, is about 85.5 Minutes, and when greatest, about 94.25 Minutes. Hence to find the least Limits of a Solar Eclipse, say;

As the Sine of the Angle E☊P = 5°46′ = 9.0020687
Is to the Side (when least) EP = 85. 5 = 1.9319661
So is the Sine of the Ang. EP☊ = 84. 14 = 9.9977966

To the Side (in Minutes) E☊ = 846.6 = 2.9276940

Hence the least Limit of a Solar Eclipse ☊E is about 14° 6′ 36″, within which there must be an Eclipse.

For the greatest Limits, say;

As the Sine of the Angle E☊P = 5°30′ = 8.9815729
Is to the Side (when greatest) EP = 94. 22 = 1.9741431
So is the Sine of the Ang. EP☊ = 84. 30 = 9.9979960

To the Side (in Minutes) E☊ = 978.5 = 2.9905662

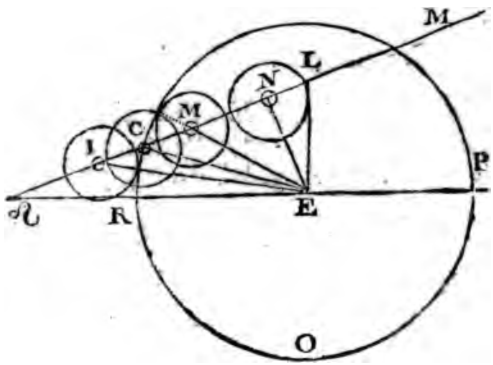
Hence the great Limits E☊ = 16° 18′ 30″, beyond which there can be no Eclipse of the Sun.

Note, The Limits are not here (as in the Lunar Eclipses) determined from the real Place of the Nodes, but from that Point in the Ecliptic, where the

the Moon's Way from the Sun interſects it, which is ſomewhat diſtant from the Node; ſo that with Regard to the Nodes, the greateſt Limits of Solar Eclipſes will be about 18°, and the leaſt about 16°, on each Side. Some Aſtronomers computing the Limits from the one Point, and ſome from the other, is the Reaſon I have calculated them from both Places.

XIII. *To calculate the Angles of Incidence, Immerſion, the Motion of Half Duration of the Whole and Central Eclipſe, and of the Penumbra within the Disk, in a Solar Eclipſe.*

IN this Figure, ☊ M is a Part of the Moon's Way, ☊ P a Part of the Ecliptic projected on the Earth's Disk R O P; I the Center of the Pen-



umbra touching the Disk; C the Center it ſelf entering the Disk; M the Penumbra totally immerſed; N E

NE the neareſt Diſtance of the Centers of the Earth's Disk and Penumbra; LE the Latitude of the Moon; and P ☊ M the Angle of the Moon's Way from the Sun; which being given, as are alſo the leaſt Diſtance, and Sum of the Semidiameters of the Penumbra and Disk, from the Tables; there may be found by *Trigonometry*,

I. *The Angle of Incidence* I E N.

As the Sum of the Semidiameters of the Penumbra and Disk —— —— } I E
Is to the Radius; So is the leaſt Diſtance of their Centers —— —— —— } N E
To the Co-ſine of the Angle of Incidence (or Exit) —— —— —— } N I E.

II. *The Immerſion of the Center* C.

As the Semidiameter of the Earth's Disk —— C E
Is to Radius; So is the leaſt Diſtance of the Centers EN
To the Co-ſine of the Angle of Immerſion of the Center —— —— —— } N C E.

III. *The Angle of Total Immerſion*, M E N.

As the Difference of the Semidiameters —— E M
Is to Radius; So is the leaſt Diſtance of the Centers NE
To the Co-ſine of the Angle of Total Immerſion of the Penumbra —— } E M N.

IV. *The Motion of Half Duration of the whole Eclipſe.*

As Radius to the Sine of the Angle of Incidence —— —— —— } I E N
So is the Sum of the Semidiameters —— E I,
To the Motion of Half Duration of the whole Eclipſe —— —— —— } I N.

V. *As*

V. *As the Radius to the Sine of the Angle of the Immersion of the Center* C E N.

So is the Semidiameter of the Disk ——— ——— C E
To the Motion of Half Duration of the Central Eclipse ——— ——— ——— C N.

VI. *The Motion of Half Duration of the Penumbra within the Discus of the Earth.*

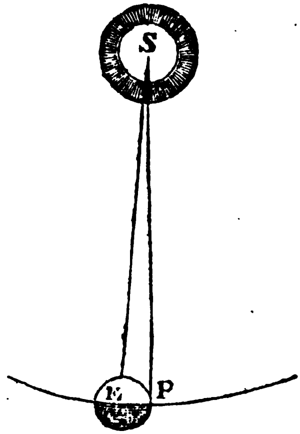
As Radius is to the Sine of the Angle of total Immersion ——— ——— M E N
So is the Difference of Semidiameters ——— M E
To the Motion of Half Continuance of the Penumbra within the Earth's Disk ——— M N.

The Times of these Motions, and consequently the Beginning, Middle, and End of the Whole and Central Eclipse, are determined as directed in *Prop.* XI. of the Lunar Eclipse.

XIV. *To determine the Distance of the Earth from the Sun.*

THIS depends altogether on the Sun's Horizontal Parallax, which is so very small as to render this Matter extremely uncertain; and tho' various Methods have been attempted to ascertain the Quantity of the said Parallax, yet none have proved infallible, the greatest Men, as well among the Moderns as the Antients, vary in their most nice and exact Observations and Accounts of the same. At present the most learned Authors, as Sir *Isaac Newton*, Dr. *Halley*, M. *Flamstead*, Dr. *Gregory*,

gory, Mr. *Whiſton*, *&c.* do agree, that it is about 10 Seconds; and therefore according to that Parallax, the Sun's Diſtance is determined as follows.



Suppoſe in the annexed Figure, S repreſent the Center of the Sun; E the Center of the Earth in its Orbit; and an Obſervation of the Sun's Place in the Horizon be made at P, and the Difference of his true Place and that be 10″, this ſhall be the Angle P S E; the Side E P alſo is known, as being the Semidiameter of the Earth; and the Angle at P is a Right one; therefore in the Right-angled Triangle P S E, there are given all the Angles and the Side E P, to find the Side E S, the Diſtance of the Centers of the Sun and Earth, by this Analogy;

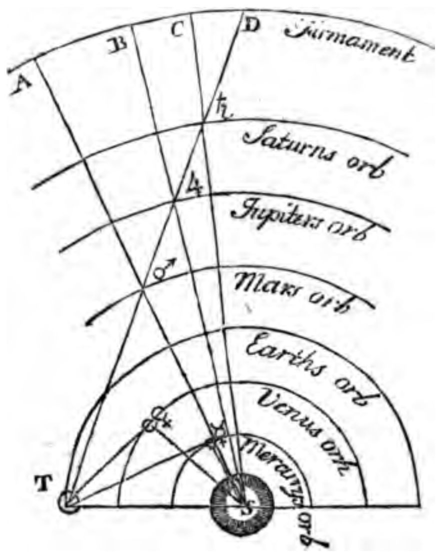
As the Sine of the Sun's Parallax	P S E	= 10″	= 5.6855626
Is to the Side	P E	= 1	= 0.0000000
So is Radius		90°	= 10.0000000
To the Diſt. of the Sun and Earth	E S	= 20627	= 4.3144374

 Then

Then multiply this Dist. in Semidiameters	=	20627
By the Miles in one Semidiameter		3982
The Prod. is the Sun's Dist. in Miles, *viz.*		82136014
Mr. *Whiston* makes this Distance	—	81000000

And some make it more, and others less; but in measuring the Distances of the other Planets, the aforesaid Distance of the Earth and Sun is divided into 100000 Parts, and the Distances of the Planets from the Earth and Sun are computed in those Parts.

XV. *To measure the Distances of the Primary Planets from the Sun, in proportional Parts of the Earth's Distance divided into* 100000.



In the Scheme adjoin'd, you observe S is the Center of the Sun and Planetary System; ☿ the Planet *Mercury*, and ♀ the Planet *Venus*, both in their greatest Elongations from the Sun; T the Earth; ♂ *Mars*, ♃ *Jupiter*, and ♄ *Saturn*, in their several Orbs;

Orbs; and which view'd from the Earth appear all in the Firmament at D, but if view'd from the Sun, *Mars* will appear in A, *Jupiter* in B, and *Saturn* in C; wherefore the Arch A D is the Measure of the Parallax T ♂ S of *Mars*, and is about 41°, when greatest of all. The Arch B D is the Measure of *Jupiter*'s Parallax T ♃ S, and is about 11 Degrees, when greatest The Arch C D is the Measure of *Saturn*'s Parallax T ♄ S, which is about 6 Degrees, when greatest.

And S T ♀ is the greatest Elongation of *Venus* from the Sun, which is about 46° 41'. And S T ☿ is the greatest Elongation of *Mercury* from the Sun, and is about 22° 46'.

These are the Definitions of the Planets Parallaxes and Elongations in general, by which we calculate their Distances from the Sun; for when they are greatest they constitute a Right Angle at ♀ and ☿, and at T for the superior Planets ♂, ♃ and ♄, and the Side T S is common in every Triangle; whence by the following Analogies are found,

I. *The Distance of* Mercury *from the* Sun.

As Radius		10.0000000
Is to the Side	T S = 100000 =	5.0000000
So is the greatest Elongat. of *Mercury*	S T ☿ = 22° 46' =	9.5878230
To the Distance of *Mer.* from the Sun	S ☿ = 38710 =	4.5878230

II. *The Distance of* Venus *from the* Sun.

As Radius	—	—	10.0000000
Is to the Side	TS	= 100000	= 5.0000000
So is the greatest Elongation of *Venus*	ST♀	= 46° 41′	= 9.8593184
To the Distance of ♀ from the Sun	S♀	= ·72333	= 4.8593184

III. *The Distance of* Mars *from the* Sun.

As the Sine of the greatest Parallax	T♂S	= 41° 00′	= 9.8171042
Is to the Side	TS	= 100000	= 5.0000000
So is Radius	—	—	10.0000000
To the Distance of ♂ from the Sun	S♂	= 152369	= 5.1828958

IV. *The Distance of* Jupiter *from the* Sun.

As the Sine of the greatest Parallax	T♃S	= 11° 05′	= 9.2839164
Is to the Side	TS	= 100000	= 5.0000000
So is Radius	—	—	10.0000000
To the Distance of ♃ from the Sun	S♃	= 520096	= 5.7160836

V. *The*

V. *The Distance of* Saturn *from the* Sun.

As the Sine of the greatest Parallax	T ♄ S	$=6^{\circ}.00' =$	9.0204526
Is to the Side	—— T S	$=100000=$	5.0000000
So is Radius	——	——	10.0000000
To the Distance of ♄ from the Sun	S ♄	$=954006=$	5.9795474

Thus the Proportion of the Distances of all the Planets from the Sun is determined, and is seen in one View, as below.

Saturn ♄.	*Jupiter* ♃.	*Mars* ♂.	*Earth* ⊖.	*Venus* ♀	*Mercury* ☿.
954006.	520096.	152369.	100000.	72333.	38710.

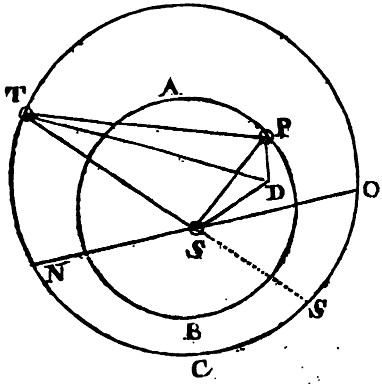
These Proportional Distances are easily turn'd into Miles by the Golden Rule, thus; As 100000 : is to the Distance of the Sun in Miles, *viz.* 82136014 : : so the Proportional Distance of any other Planet : to its Distance from the Sun in Miles.

XVI. *To calculate the Geocentric Place, Latitude, and Distance of a superior or an inferior Planet, as* Venus *or* Mars.

SUppose you have (as you may have for any Point of Time) from the Astronomical Tables, the Heliocentric Place and Distance of the Planet *Venus*, for Instance, from the Sun; the Place and Distance of the Earth for the same Moment being also

alſo given from the ſaid Tables, we can eaſily find the Planet's Place, Latitude, and Diſtance with reſpect to our ſelves.

In the following, let S be the Sun; A P B the Orb of an inferior Planet; N S O the Line of its Nodes; P its Heliocentric or true Place in its Orb; T O C the Earth's Orb, and T its Place therein.



Then, from the Tables there is given the Diſtance from the Earth and Sun T S; and the Curtate Diſtance of the Planet S D; alſo is Heliocentric Latitude P S D; laſtly, there is given the Angle of Commutation T S D, the Difference between the Place of the Earth T (or Sun S) and the Planet P, as ſeen from the Sun. Theſe Things being thus given, we can find,

I. *The Elongation of the Planet from the* Sun, STD.
II. *The Parallax of the* Orbis Magnus, TDS.
III. *The Geocentric Curtate Diſtance,* ——— T D.
IV. *The Geocentric Latitude of the Planet,* PTD.

For, in the Oblique Triangle T S D, there are given two Sides T S and S D, and the Angle included T S D, to find the other two Angles S T D the Elongation, and T D S the Parallax of the Orb of the Earth, by *Caſe* III. of *Oblique Triangles.*

Then

Then, by *Caſe* I. in the ſame Triangle, you can find TD the Curtate Diſtance of the Planet from the Earth.

Laſtly, in the two Triangles DSP and DTP, Right-angled at D, the Tangent of the Angle DSP is to the Tangent of DTP, as TD is to DS; But as TD is to DS, ſo is the Sine of the Angle of Commutation TSD to the Sine of the Elongation STD. Wherefore it will be,

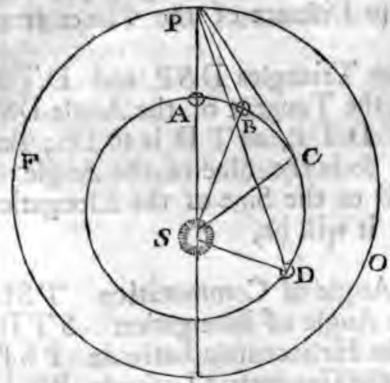
As the Sine of the Angle of Commutation TSD,
Is to the Sine of the Angle of Elongation STD;
So is the Tang. of the Heliocentric Latitude PSD,
To the Tangent of the Geocentric Latitude PTD.

This Scheme is adapted indeed only to an inferior Planet, but the Method and Proportions are equally the ſame in any ſuperior one; only this is to be minded, that in finding the two Angles of the Parallax and Elongation, the greater Angle in the ſuperior Planets is the Elongation; but in the inferior ones, it is the Parallax of the Orb, *et e Contra*, for the Angle which is leaſt.

XVII. *To determine the Orb of the Earth, or to find its Axis's, Poſition, and Eccentricity.*

THIS grand and fundamental Propoſition of all true Aſtronomy is performed by a very curious and accurate Method (among others) invented by that preſent great Prince of Aſtronomers Dr. *Edmond Halley*, and is thus;

Let



Let POF be the Orb of a Planet, suppose of *Mars*, because his Orb is the most eccentric; and ABCD the Orb of the Earth; let S be the Sun.

Now, let the true Time and Place of the Ecliptic be observed when *Mars* is in Opposition to the Sun, that is, when the Sun S, the Earth A, and the Planet P are in one Right-line PAS; which must be in, or (when *Mars* has any Latitude) reduced to the Plane of the Ecliptic. Since *Mars*'s periodical Revolution is in 687 Days, after that Time *Mars* will return to the Point P again, and the Earth in that Time having made one Revolution (365 Days) and almost another (322 Days) is arrived, suppose, at B; where the Planet (*Mars*) will appear in the Line BP, and the Sun in the Line BS; Now, by observing the Places of the Sun and *Mars*, or the Angle of Elongation PBS, the Angle ASB being also known, we have, in the Oblique Triangle PBS all the Angles, and supposing the Distance of the Planet PS to be divided into 100000 Parts, we can thence, by *Case* I. of *Obl. Triangles*, find the Side or Line SB in those Parts, and hence the Length and Position of the Line SB is known.

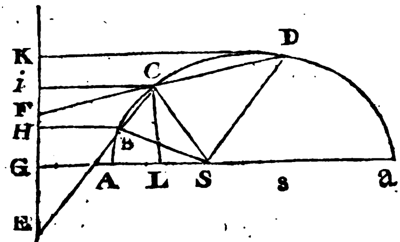
Let *Mars* perform another Revolution to P, and then suppose the Earth in C; thus we shall have the Length and Position of the Line SC, as before.

After

After the ſame Manner when the Planet has finiſhed another Period, the Earth will be in D; and therefore the Poſition and Length of the Line S D is given.

Thus, Having three Lines meeting in the Focus of an Ellipſis, all given in Length and Poſition, the Tranſverſe Axis, Diſtance of the Foci, and conſequently the whole Theory of the Planet moving in this Elliptic Orb, may be known, or found by *Trigonometrical* Calculation, as in the following Propoſition.

XVIII. *Having three Lines meeting in the Focus of an Ellipſe, all given in Length and Poſition, to find the Length of the Tranſverſe Diameter, and its Poſition, and Diſtance of its Foci.*



LET SD, SC and SB be the three Lines given in Magnitude and Poſition; Draw DC and BC, and produce them; ſo that it may be SD : CS :: DF : CF. And CS : BS :: Ck

Join FE, on which let fall the Perpendicular SG, which will give the Poſition of the Axis, as being Part thereof. Draw the Lines DK, Ci, and BH parallel to GS; and cut SG ſo in A, that it may be KD : SD :: GA : SA :: Ga : Sa, and make sa = SA; then ſhall

Gg the

the Points A and a be the Apſides of the Orbit, whoſe Foci are S and s, and Tranſverſe Axis A a; the Quantity of which Lines are thus inveſtigated by *Trigonometry*.

In the DSC having the DS and CS, and the Angle DSC; we can find the Side DC, and the Angles SCD and SDC, by *Caſe* III. of *Oblique Triangles*. In the ſame Manner we can find in the Triangle BSC, the Side BC, and the Angles SBC and SCB; and becauſe it is SD : CS :: D : CF, and DC being known, CF will alſo be known; in like Manner we ſhall alſo have BE and CE; but we have the Angle BCD, and therefore FCE is Complement to 180°. In the Triangle FCE we have the Sides FC and CE, and the Angle FCE, and therefore can find the Angle FEC, and ſo its Complement to a Right-angle iCE; to which add the known Angle SCB, we have the whole Angle SCi; and becauſe of the parallel Lines Sa and Ci, the Angle CSa is equal to the Angle SCi. Having therefore the Angle CSa, we have the Poſition of the Tranſverſe Axis.

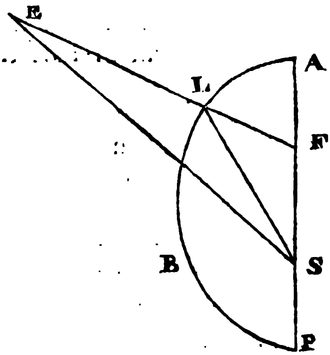
In the Right-angled Triangle ECi having EC, and the Angle E, we can find Ci, and therefore the Proportion of CF to Ci.

In the Triangle CSL, right-angled at L, there is given the Angle CSL (the Complement of CSa) and the Side CS, hence we ſhall find SL, to which adding LG=Ci, we have the Whole SG: and becauſe CS and Ci are known, Let it be made CS : Ci :: SA : AG :: Sa : aG :: Ss : Aa; and thus we ſhall have the Apſides of the Ellipſe A, a; and the Diſtance of the Foci Ss, which were to be found.

This

This Distance of the Foci, or Centers of the Ellipsis, is the Double of that, which in the Planetary Orbs, is called the Eccentricity. And thus by these two last Propositions the Axis of the Earth's Orb, its Position, and Eccentricity, are readily determined, by Observation and *Plain Trigonometry.*

XIX. *By having the Eccentricity and Mean Anomaly, to find the Prosthaphæresis, Equation, or true Anomaly of any Planet.*



LET ABP be one Half of the Ellipse, which the Planet describes; AP the Line of the Apsides; S the Focus in which the Sun is placed; F the other upper Focus which is the Center of the Mean Motion, according to Bishop *Ward's* elegant and easy Hypothesis. Let L be the Place of the Planet in its Orb; then shall the Angle AFL be the given Mean Anomaly, and ASL the Co-equated, or True Anomaly; in order to find which by *Trigonometry* proceed thus; Produce FL to E, so that FE=AP (=FL+LS, by the Nature of an Ellipsis.) Hence LE=LS, and therefore the Triangle LSE is an Isosceles one, and so the Angle E=ESL; But the Angle SLF =E+ESL,

or Double of either (by *Theor.* V.) Therefore in the Triangle E F *S*, having FE and FS, and the Angle E F S the Complement of the mean Anomaly A F L to 180° we can find the Angles E and E S F, by *Case* III. of *Oblique Triangles*, thus;

As $\frac{EF+FS}{2}$: $\frac{EF-FS}{2}$, that is, AS : SP ::

Tangent ½ A F E : Tangent of $\frac{ESF-E}{2}$ =FSL;

which Analogy put into Words is thus;

As the Aphelium Distance ——— ——— A S,
Is to the Perihelium Distance ——— ——— S P;
So is the Tang. of half the Mean Anomaly A F L,
To the Tangent of half the True Anomaly A S L.

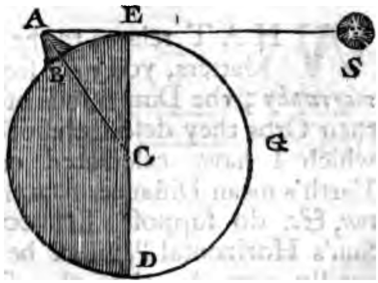
The Difference therefore between the Mean Anomaly and the True, is the Angle SLF; which, because if added to, or taken from the Mean Anomaly, it gives the True Anomaly, is therefore called the Prosthaphæresis, or Equation of the Orb, and is ready calculated to every Degree of the Mean Anomaly in the Astronomical Tables.

This Method of Bishop *Ward's* is but an ingenious Approximation to the True Anomaly, not the Truth it self; therefore *Bullialdus* has given a very considerable Correction thereof, which may be seen in Dr. *Gregory's*, Dr. *Keil's*, and Mr. *Whiston's* Astronomical Works.

XX. To

XX. *To measure the Height of the Mountains in the Moon.*

THE famous Astronomer *Ricciolus* affirms, that on the 4th Day after the New Moon he has obferv'd the Top of the Hill call'd *St. Katharine*, or Mount *Sinai*, to be illuminated, and that it was diftant from the Confines of the Lucid Surface about a 16th Part of the Moon's Diameter, or an 8th Part of the Semidiameter.



Let BGD reprefent the Moon; A the Top of a Mountain thereon; CE be the Semidiameter of the Moon, which let be 8, then will AE be 1; therefore in the Triang. AEC Right angled at E, there is given two Sides $AE=1$, and $EC=8$, to find the Side AC, which by *Method* VIII, is thus; The Squares $AEq+CEq=1+64=65$; but $AC=\surd 65 =8.062$; Now $AC-BC=0.062=AB$ the Height of the Mountain A. But fince (by *Propofition* IV.) the Semidiameter of the Moon is 1079.5 *Englifh* Miles, we fhall find the Miles in AB, by this Analogy; As 8 : 1079.5 : : ,062 : 8 $\frac{3}{10}$ *Englifh* Miles, the Height of the Mountain fought.

But, confidering the fmall Bulk of the Moon compared with the Earth, it feems not likely that a Mountain in the Moon fhould be 3 Times as high as the higheft on Earth; 'tis therefore to be fuppo-

fed

fed that *Ricciolus's* Obfervations were defective; fince *Hevelius*, in his *Selenography* reckons them in General fcarcely fo high as the Mountains on the Earth. See more in *Derham's* Aftro-Theology *B.* V. *Chap.* 2. in the Notes and the Books he there quotes, or refers to.

XXI. *To determine the Plane, Square, and Cubic or Solid Meafures of the Planets, and their Orbs.*

WHAT relates to the Earth, concerning thefe Matters, you may fee in the Chapter of *Cofmography*; the Dimenfions of the other Planets, and their Orbs they defcribe about the Sun, here follow; which I have calculated on the Suppofition the Earth's mean Diftance from the Sun be (as Mr *Whifton, &c.* do fuppofe) 81000000 Miles; tho' if the Sun's Horizontal Parallax be indeed 10″, as is generally agreed, then the faid Diftance will be 82136014, as by *Prop.* XIV. does appear. However the other being a round Number, and not greatly different, the Dimenfions of the Planets according to it are here ftated, *viz.*

Of MERCURY.

	Miles.
His Diameter	2460
The Circumference of his Body	7724
The Superficies in Square Miles	19000804
The Solidity in Cubic Miles	7793273000
The Diameter of his Orb	64000000
The Circumference of his Orb	201024000

Of

Of VENUS.

	Miles.
Her Diameter — — —	7906
The Circumference of her Body —	24823
The Superficial Content — —	196238000
The Solidity — — — —	258445900000
The Diameter of her Orb —	118000000
The Circumference of her Orb —	370636000

Of the MOON.

Her Diameter — — — —	2775
Her Circumference — — —	6829
The Superficial Content — —	14855440
The Solidity — — —	5386333000
The Diameter of her Orb — —	477840
The Circumference of her Orb —	1500418

Of MARS.

The Diameter — — —	4444
The Circumference — — —	13960
The Superficies — — —	62032900
The Solidity — — —	45966600000
The Diameter of his Orb — —	246000000
The Circumference of his Orb — —	773686000

Of JUPITER.

The Diameter — — —	81155
The Circumference — — —	254908
The Superficial Content — —	20780000000
The Solid Content — —	281042300000000
The Diameter of his Orb — —	848000000
The Circumference of his Orb —	2662280000

Of

Of SATURN.

	Miles.
His Diameter ——— ———————	67870
His Circumference ——— ———	213112
The Superficial Content—————	14468430080
The Solidity—— ——— ——	163637700000000
The Diameter of his Orb——— ———	1554000000
The Circumference of his Orb ———	4881891000

	Mer.	*Ven.*	*Earth.*	*Moon.*	*Mars.*	*Jup.*	*Sat.*	*Sun.*
The Density of the Plan.	**	**	3.87	7.00	**	0.76	0.60	1.00
The Gravity	**	**	1.00	0.34	**	2.00	1.28	24.40
The Propor. of Magnit.	78.	2584.	2644.	53.	459.	2810423.	1636377.	
Prop. of Heat	600.	200.	100.	100.	40.	37.	11.	

The Heat of the Sun being ſuppoſed 4500000.

Thus I have finiſhed the Modern Ὀυρανομετρία, or Dimenſions of the Heavens, and Heavenly Bodies therein; which you ſee is altogether performed by *Plain Toigonometry*.

This Ὀυρανοσκοπία, or Contemplation of the Heavens, the Sun, the Moon, and the Stars, ſhould lead us to admire the infinite Power and Wiſdom of that Great Being who made not only this, but (probably) numberleſs Syſtems of other Planetary Worlds, diſpoſed thro' the univerſal Space.

CHAP. V.

CHAP. IV.

Plain Trigonometry *applied to* Fortification; *Shewing how the Quantity of the Sides and Lines of any Fort are determined by* Trigonometrical *Calculation.*

FORTIFICATION is an Art that requires no ſmall Skill in *Plain Trigonometry*, as will appear by the Sequel of this Chapter.

Fortification is called *Military Architecture*, as it ſhews how to fortify a Place by conſtructing Ramparts, Parapets, Moats, and other Bulwarks of Defence, to the End that a few Men in Gariſon, thus circumſtanced, might be able to defend and ſecure themſelves, and the Place from the Attacks of a numerous Army of the Enemy without.

Fortification is either Regular or Irregular; that is, a Regular Fortification which is built on the Plan of a Regular Polygon, the Sides and Angles of which are all equal; as a *Pentagon*, *Hexagon*, &c.

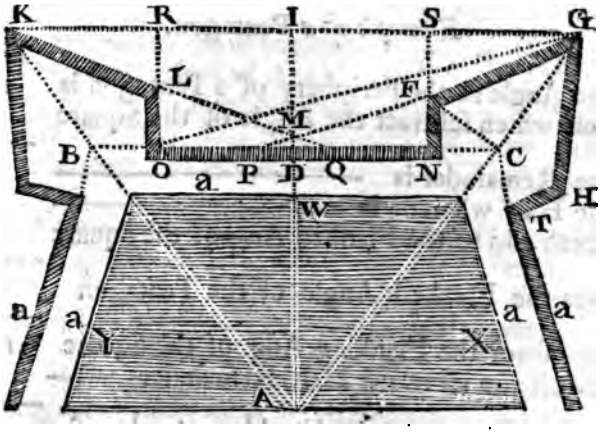
Irregular Fortification is that whoſe Sides and Angles are not uniform and equal, nor equi-diſtant from each other.

But a more particular Description of each Part, (about which the *Trigonometer* is more immediately concern'd) take as follows. See the *Figure*.

I. A Fort is a Piece of Ground environed with a Rampire or Wall, and a Ditch to impede the Assaults of an Enemy; as the Pentagonal Fort	YWXA.
II. A Rampire is a Wall of Earth enclosing the Place fortified, the Foundation whereof is	a a, a a, a.
III. The Curtain is the Side	O N.
IV. The Bulwark is the Part	NFGHT.
V. The Front of a Bulwark	FG or KL.
VI. The Flank of the Bastion or Bulwark is	FN or LO.
VII. The Gorge of a Bulwark, is the Space between its two Flanks, as	N T.
VIII. The Gorge Line is the Line	N C.
IX. The Head Line of the Bastion is	C G.
X. The Shoulder of the Bastion is	F, or L.
XI. The Flanked Angle, or Diamond Point	G.
XII. The Line of Defence fixed or Fichant, is	O G.
XIII. The Inward Flanking Angle is	S G F.
XIV. The Outward Flanking Angle is	G M K.
XV. The Angle of the Shoulder	N F G.
XVI. The False Bray	B C.

There are various other Terms used in this Art, but these are sufficient to answer my Purpose; which is only to shew the Necessity of the Doctrine of *Plain Trigonometry* for all such as would have Notion or Skill in *Fortification*.

The



The Quantity of the Angle at the Periphery of any Polygon, is called (by Authors on this Subject) the Angle of the Polygon; and Half the Quantity of the Angles at the Center (which is always) 90 Degrees, they call the Angle of the Square; also the Flanked Angle of the Square being always 60 Degrees, and the outward Flanking Angle being 150 Degrees; there follows this general Rule for finding the same Flanked and Flanking Angles in any other Regular Polygon; *viz.*

The RULE.

Subtract the Angle of the Square 90° from the Angle of the Polygon; Half the Remainder add to the Flanked Angle of the Square 60°, the Sum is the Flanked Angle of the Polygon proposed.

Also subtract the Half-Remainder from the Flanking Angle of the Square 150°, and there will remain the Flanking Angle of the given Polygon.

Example

Example of a Pentagon.

	Deg.
The Angle at the Periphery of a Pentagon is	108
From which subtract the Angle of the Square	90
The Remainder is ————————	18
The Half whereof is ————————	9
Which add to the Flanked Ang. of the Square	60
Gives the Flanked Angle of the Pentagon	69
Then from the Flanking Ang. of the Square	150
Subduct the aforesaid Half Remainder ——	9
There will remain the Flanking Angle of the given Pentagon ————————	141

Now, the Flanked Angle of the Bulwark being known, we can thereby come at the Knowledge of all the other Angles necessary to be known, in the Method following.

		° ′
In the foregoing Figure, the Angle at the Center of the Pentagon is	BAC	= 72.00
The Half thereof is————————	CAD	= 36.00
The Complement of which to 90° is	DCA	= 54.00
The Angle at the Bulwark is ——	FGH	= 69.00
The Half thereof is —— ——	FCG	= 34.30
Which subtracted from SGC=DCA there will remain the Inward Flanking Angle ————	SGF	= 19.30
Therefore its Complement is ——	SFG	= 70.30
Which subtract from two Right-Ang.		180.00
Leaves the Angle of the Shoulder.	NFG	= 109.30

Also

Also 2 SFG=2IMG, is equal to the Outward Flanked Angle } KMG = 141.00 (° ′)

Lastly, from two Right-angles ——— 180.00

Subtract Half the Ang. of the Polyg. BCA = 54.00

There remains the Angle ——— DCG = 126.00

And thus from the Flanked Angle of any other Polygon given, are the other Angles to be determined. And tho' there is no Necessity that the Angles of a Fort should be exactly such as are found by the above Method, but may be something more or less as the Place or other Occasions may require; yet supposing them such, in the Pentagon, the Quantity of the Sides are to be determined by *Trigonometrical* Calculation, as in the subsequent Method.

In Order to this, there must be given — *Feet.*

I. The Curtain, which suppose —— ON = 420

II. The Front of the Bulwark —— FG = 280

Whence the other Sides are thus found.

I. *In the Right Triangle* SGF, *by Case* III.

As Radius ——— ——— 10,0000000

Is to the Front of the Bulwark ——— } FG = 280 = 2.4471580

So is the Sine of the Flanking Angle } SGF=19°30′= 9.5234953

To the Line ——— SF = 93.47 = 1.9706533

II. *In*

II. *In the same Triangle, and by the same Case.*

As Radius — — — — —		10.0000000
Is to the Front of the Bulwark — —	FG = 280 =	2.4471580
So is the Sine of the Angle — —	SFG = 79° 30′ =	9.9743466
To the Line ———	SG = 263.94 =	2.4215046
Add Half the Curtain	S I = 210.00	———

The Sum is the Line——IG = 473.94
The Double whereof is KG = 947.88 the Side of the outer Pentagon, or Distance of the Diamond Points G, K.

III. *In the Right-Triangle* I A G, *by Case* II.

As the Sine of the Angle	IAG = 36° 00′ =	9.7692187
Is to the Side ———	I G = 473.94 =	2.6757200
So is Radius ———		10.0000000
To the Side, or Line——	AG = 806.3 =	2.9065013

IV. *In the same Triangle, by Case* I.

As the Sine of the Angle	IAG = 36° 00 =	9.7692187
Is to the Side — —	I G = 473.94 =	2.6757200
So is the Sine of the Ang.	AGI = 54.00 =	9.9079576
To the Side or Line ——	IA = 652.3 =	2.8144589

V. In

V. *In the Oblique Triangle* FCG, *by Case* I.

As the Sine of the Ang.	FCG=86°00′=		9.9989408
Is to the Front of the Bulwark ———	FG= 280 =		2.4471580
So is the Sine of the Ang.	FGC=34°30′=		9.7531280
To the Line ———	FC=158.98=		2.2013452

VI. *In the same Triangle, and by the same Case.*

As the Sine of the Angle	FCG=86°00′=	9.9989408
Is to the Front ———	FG= 280 =	2.4471580
So is the Sine of the Ang.	GFC=59°30′=	9.9353204
To the Line ———	CG =241.44=	2.3835376
Which subduct from———	AG =806.31	
There remains———	AC =564.87	

VII. *In the Right Triangle* FCN, *by Case* III.

As Radius ———		10.0000000
To the Line ———	FC=158.98=	2.2013452
So is the Sine of the Ang.	FCN=40°00′=	9.8080675
To the Flank Line ———	FN=102.19=	2.0094127
To which add the Line	SF= 93.47	
The Sum is the Line———	ID=195.66	
Which subtract from ———	AI=652.32	
There remains the Line	AD=456.66	

VIII. *In*

VIII. *In the same Triangle, and by the same Case.*

As Radius ————————		10.0000000
Is to the Side ——	FC=158.98=	2.2013452
So is the Sine of the Ang.	CFN=50°00′=	9.8842540
To the Gorge Line——	NC =121.78=	2.0855992
Add Half the Curtain	DN =210.	
The Sum is the Line——	DC =331.78	
The Double of which is	BC =663.56	

IX. *In the Right Triangle* FPN, *by Case* II.

As the Sine of the Ang.	FPN=19°30′=	9.5234953
To the Flank ———	FN =162.19=	2.0094127
So is the Sine of the Ang.	PFN= 70.30=	9.9743466
To the Line—————	PN =288.58=	2.4602640
Which sub. from the Curt.	NO=420.	
There remains the Side	OP =131.42	

X. *In the Right Triangle* ROG, *by Case* IV.

As the Line (SG+ON=)	RG=683.94=	2.8350200
Is to the Side———	OR=195.66=	2.2914900
So is Tangent Radius ————		10.0000000
To the Tang. of the Ang.	RGO=15°58′=	9.4564700

XI. *In the same Triangle, by the same Case.*

As the Sine of the Angle	RGO=15°58′=	9.4394560
Is to the Side or Line	RO =195.66=	2.2914900
So is Radius ————————		10.0000000
To the longest Line of Defence ———	OG=711.4=	2.8520340

Thus

Thus you have delivered a Method for finding the Quantity of the Lines, Sides, and Angles of any regular Fortification; and herein the young Student will see farther the great Use and Excellency of *Plain Trigonometry.*

CHAP. V.

Plain Trigonometry *applied to the Doctrine of* Projectiles, *or Art of* Gunnery; *shewing the Calculation of the Impetus, Direction, Amplitude, Height,* &c. *of the Projections, or Randoms, of Bullets and Bombs shot or thrown from Pieces of Ordnance.*

THE Art of Gunnery altogether depends on the Doctrine of Projections, and its chiefest Propositions are performed by *Trigonometrical* Calculations.

The Line of Motion, which a Body projected describes, abstracting from the Resistance of the Medium, is the Curve of a Parabola, as has been of late proved by many Mathematicians, and particularly by Sir *Isaac Newton* in his *Math. Principles of Philosophy.* Which Line is also described by every descending Body.

In the Doctrine of Projection, or Projected Bodies, we are to consider principally these following Things;

I. The Impetus or Force wherewith the Body is projected from any given Point.
II. The Direction of the Motion of the projected Body, which is either Parallel to the Horizon, or obliquely inclined thereto, above or beneath it.
III. The Amplitude of the Projection, or Distance to which a Body is cast, in any Direction.
IV. The Height or Altitude of the Projection, or the highest Point of the Curve it describes above the Horizon.

With Respect to the Art of Gunnery, the above-mentioned Particulars are to be understood thus;

The Impetus is the Force of the Gunpowder, whereby the Bullet or Bomb is violently projected from the Ordnance.

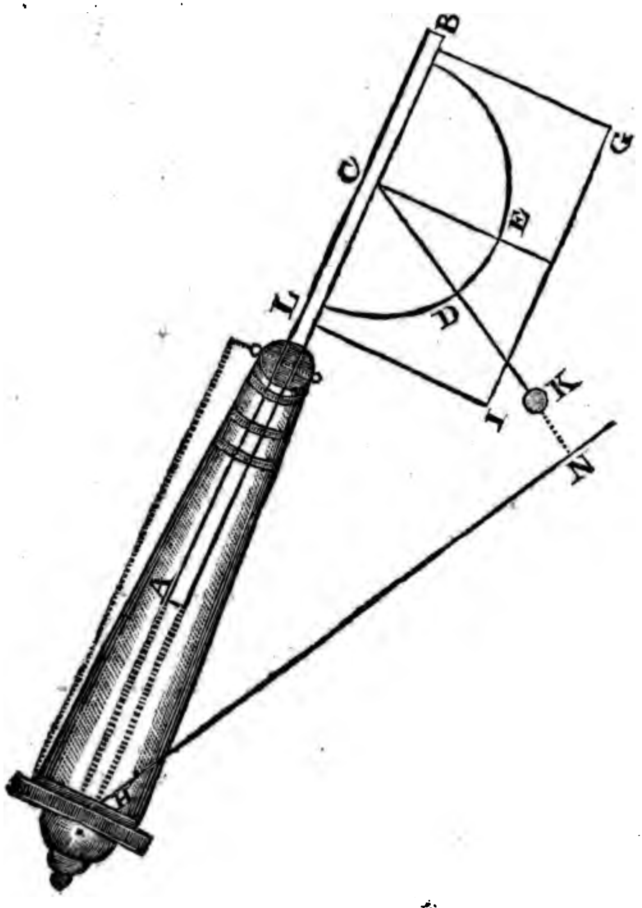
The Direction is the Position of the Cannon, either parallel to, or elevated above the Horizon, in any given Angle.

The Amplitude of the Projection, is the Range, or Random (rather) of the Range, or Line of the Bullet's Course, before it grazes.

The Height of the Projection is the Height of the Range, for the Projection is call'd the Range, in this Art.

Concerning the Direction and Elevation of a Piece of Ordnance, these following Particulars are to be noted.

The Direction of a Cannon is to be reckoned the same with the Direction of its Concavity; for by the



the Firing of the Gunpowder the Bullet is emitted according to the Concavity of the Cannon or Mortar; and was it not for Gravity, it would proceed in that Right-line produced; so that that Right-line is the Direction of the Ordnance.

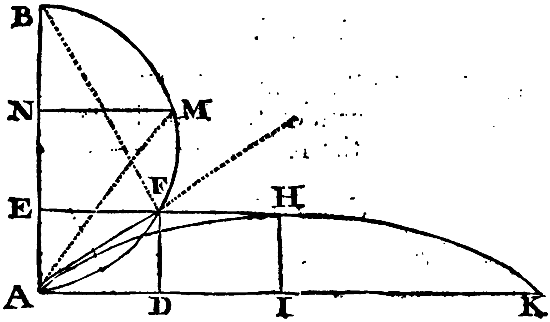
 When

When Walls are to be batter'd, or any Thing to be done, where a great Impetus is required, and the Mark is not above 200 Paces off, and the Engine sufficiently; In such Shots as these, the Direction of the Cavity, proper Quantity of Powder, and Weight or Size of Balls, are the only Requisites to strike the Mark Point Blank, or directly.

But, since Citadels, or the Town to be bomb'd, cannot most commonly, by reason of its too great Distance, be hit by a direct Level; the Engine must be elevated in a given Angle to the Horizon; For which Purpose, there is used a Ruler ACB, whereunto is fixed a graduated Semicircle LEB in the Parallelogram LIGB, from the Center of which hangs a Plummet K. The Ruler must be inserted into the Mouth or Cavity of the Piece, and held parallel to the Axis thereof; and thus the Ordnance is to be raised or depressed, till the Perpendicular CK cuts the Degrees of Elevation required in the Point D, reckoning the Degrees from E L. For 'tis evident by Inspection, that the Angle ECD is equal to the Angle CHN the Elevation of the Engine; because the Angle DCL is the Complement of both to a Right-angle.

In order to reduce the Doctrine of Projection of Heavy Bodies to a Methodical Calculation, and apply it at the same Time to the Art of Gunnery; we must, instead of the Force of the Powder, consider the Perpendicular Height to which the given Charge of Powder is sufficient to throw the Ball; and having the Perpendicular Height of any Impetus of the Powder, this may be represented by a Right-line, and therefore the Calculation may be made with Ease, as follows;

In



In the Figure above, let A be the Mouth of a Cannon whence a Bullet is to be projected in the Direction AE, and let AB be the Perpendicular Height to which the Bullet might be thrown by any given Charge of Powder; then shall the Bullet, instead of proceeding in the Direction AE, describe the parabolical Arch AHK whose Height above the Horizon is HI, and its Amplitude AK. Now AD is $\frac{1}{4}$ of AK, and DF=HI, and the Angle FAD is the Angle of Elevation of the Engine; but because the Angle AFB is in a Semicircle, it is therefore a Right one; and by Reason of Similar Triangles, the Angle DAF = AFE = EBF; and therefore, AD=EF, is equal to the Sine of double the Angle of Elevation, or 2ABF. Wherefore as AD is one Fourth of the Amplitude, 'tis evident, when the Angle of Elevation is half a Right one, that then AD = HI = $\frac{1}{2}$AB = $\frac{1}{4}$AK. That is, that the Height of the Range HI would be equal to ($\frac{1}{2}$AB) half the Perpendicular Height, and equal to ($\frac{1}{4}$AK) a Quarter of the Amplitude; whence also the Amplitude is equal to (2AB) twice the Perpendicular Height in such a Case. Whence 'tis easy from the foregoing Premises to deduce the following *Canons* for Calculation.

Canon

Canon I.

As Radius : is to the Tangent of the Elevation FAD,
: : So is AD, one Fourth of the Amplitude AK,
: To the Height of the Range —— —— HI.

Canon II.

As the Sine of the Angle of Elevation —— FAD,
: Is to the Radius,
: : So is DF : AF; and then
: : So is AF : AB the Height perpendicularly.

Canon III.

As Radius : to the Sine of the An. of Elevation ABF,
: : So is the Time of the Perpendic. Projection AB,
: To the Time of the Projection in —— AF.

Canon IV.

As the Sine of Double the Angle of Elevation, or —— —— —— } 2KAF,
: Is to the Sine of Double the Angle of any other Elevation —— —— } 2KAM
: : So is the Amplitude AK made in the Direction —— —— —— } AF,
: To the Amplitude in the Direction —— AM.

Canon V.

As the Versed Sine of Double the Elevation KAF,
: Is to the Versed Sine of Double the Elev. KAM;
: : So is the Height of the Range in the Direction —— —— —— } AF, *viz.* AE.
: To the Height of the Range in the Direction —— —— —— } AM, *viz.* AN.

Canon

Canon VI.

As the Sine of the Angle of Elevation —— KAF,
: Is to the Sine of the Ang. of the Elevation KAM,
: : So is the Time of the Projection in —— AF,
: To the Time of the Projection in —— AM.

By these Six *Canons*, all the most useful Part of the Military Art, or the Propositions of Gunnery, so far forth as they come under a *Trigonometrical* Consideration, are readily performed; as will appear by the Operations of the ensuing Propositions.

Proposition I.

Admit a Ball be thrown to the perpendicular Height of 909,2 Paces with a given Charge of Powder, how far will the same Charge throw it in the Direction AF, or Elevation of the Angle FAD = 17° 45′? *Quere* also the Height of its Range HI, and greatest Amplitude?

For the Amplitude, say;

As Radius —— ——	——	10.0000000
Is to the Half Perpen. Height =	454.6	=2.6576294
So is the Sine of the Double Angle of Elevation ——	35° 15′	=9.7612128
To a Fourth of the Amplitude	262.4	=2.4188422
	4	

The whole Ampl. therefore is 1049,6 Paces, or 1 Mile *fere*.

For

For the Height of the Range, say ;

As Radius — — — —	10.0000000
To the Tang. of the Elevation 17° 45′ =	9.5052891
So is a Fourth of the Amplit. 262.4 =	2.4188422
To the Height of the Range in Paces — — — } 84 =	1.9241313

For the greatest Amplitude, say ;

As the Sine of Double the Elevation — — } 35° 15′ =	9.7612128
Is to the Sine 90°, or greatest Sine =	10.0000000
So is the Random already found, 1049.6 =	3.0209755
To the greatest Rand. or Ampl. 1818. =	3.2597627

Proposition II.

Suppose a Shot with a Mortar upon 18 *Degrees of Elevation throws a Bomb one Mile, or* 1056 *Paces ;* Quere *the Height of the Perpendicular Shot, and the Height of the Range ?*

For the Height of the Range, say ;

As the Radius — — —	10.0000000
Is to the Tang. of the Elev. 18° 00′ =	9.5117760
So is a Fourth of the given Ran. 264 =	2.4216039
To the Height of the Range 85.7 =	1.9333799

For

For the Perpendicular Height.

As the Sine of the Elevation	18° 00′ =	9.4899824
Is to Radius	— — —	10.0000000
So is the Height of the Range	85.7 =	1.9333799
To a Fourth Number —	277.5 =	2.4433975
And so is that, to the Per. Height	898.2 =	2.9534151

Proposition III.

Suppose a certain Charge of Powder, sufficient to throw a Ball to the Perpendicular Height of 800 *Paces, should cast a Ball to the Distance* 1260 *Paces*; Quere *the Elevation and Height of the Random?*

For the Elevation, say;

As Half the Perpend. Height	400 =	2.6020600
Is to a Fourth of the given Rand.	315 =	2.4983105
So is Radius	— — —	10.0000000
To the Sine of Double Elev.	=51° 57′ =	9.8962505

The Half of which is 25° 58′; but that is also the Sine of 128° 3′; the Half of which is 64° 1′; therefore the Elevation may be either 25° 58′, or 61° 1′, indifferently.

For the Height of the Random.

As the Radius	— — —	10.0000000
Is to the Tang. of the Elevat.	25° 58′ =	9.6875402
So is a Fourth of the given Amp.	315 =	2.4983105
To the Height of the Random	153.4 =	2.1858507

If

If the Height in the Elevation 64° 1′ be required, say;

As the Versed Sine of Double the first Elevation 51° 57′	3836	= 3.5838786
Is to the Versed Sine of Double the second Elevation 128° 3′	16163	= 4.2084914
So is the Height in the 1st Elevat.	153.4	= 2.1858507
To the Height in the 2d Elevat.	646.3	= 2.8104635

This might also be found (and that most easily) as the first Height; but I used this Method of Versed Sines for Variety's Sake.

Proposition IV.

Suppose a Shot made upon 45 Degrees of Elevation, continues 12 Seconds of Time in the Air; 'tis demanded *How long it will continue in the Air when shot from 75 Degrees of Elevation?*

Say, As the Sine of the Elevation	45°	= 9.8494850
Is to the Sine of the Elevation	75	= 9.9849438
So is the Time	12″	= 1.0791812
To the Continuance in the Air req.	16″3	= 1.2146390

For the Time of the Perpendicular, say;

As the Sine of the Elevation of	45°	= 9.8494850
Is to Radius		10.0000000
So is the Time	12″	= 1.0791812
To the Time of the Perpen. Shot	16″9	= 1.2296962

From

From these Propositions, and their particular Examples, I suppose 'tis sufficiently evident how necessary and useful the Art of *Plain Trigonometry* is in the Art of *Gunnery*, in Casting Bombs, Fusees, and other Instruments of Destruction in Times of War, and in Cases of Necessity, so as to effect the designed Purposes with Certainty and Success. And this is all I intend, leaving what else concerns this terrible Affair to those who have more Knowledge therein than my self.

CHAP. VI.

Plain Trigonometry *applied to* Mechanics; *in computing the Directions, Celerities, Momenta, and Forces of moving and striking Bodies.*

IN order to understand the Doctrine of Motion, the following Definitions are necessary to be premised.

Definitions.

I. *Motion* is a continual and successive Change of Place.

II. *Celerity* is an Affection of Motion, whereby a Body in Motion passeth over a given Space in a given Time.

III. *Rest* is the Permanence, or Continuance of any Body in the same Place.

IV. *Place* is the Space taken up by any Body; and is two-fold, absolute or relative.

V. *Absolute Place* is that Part of the immoveable Space which a Body takes up.

VI. *Relative Place* is the Situation of a Body in respect of other Bodies, and can only be discerned by our Senses; and is changeable, while absolute Place remains the same; and *e contra*.

VII. *Absolute Motion* is a Change of absolute Place, and its Celerity is measured by absolute Space.

VIII. *Relative Motion* is the Change of relative Place, and its Celerity is measured by relative Space.

IX. *Absolute Rest* is the Permanence of a Body in the same absolute Place.

X. *Relative Rest* is the Permanence of a Body in the same relative Place.

XI. The *Direction of Motion* is a right Line, according to which the moving Body tends to any certain Point or Place.

XII. *Equable Motion* is that which is performed in every Part of Space passed over, with equal or the same Celerity.

XIII. *Accelerated Motion* is that whose Celerity or Velocity continually increaseth.

XIV. *Retarded Motion* is that whose Velocity is continually diminished.

XV. The *Momentum*, or *Momenta*, is the Quantity of Motion, or the Quantities of Motion in moving Bodies compared, which is compounded of the Quantity of Matter and Celerity of Motion, in any Body.

XVI. A *Power* is any Force impressed, or acting on any Body, to change its State, either of Motion or Rest.

XVII. *Gravity* is a Force tending downwards, or whereby Bodies tend towards the Center of the Earth.

XVIII. A *Centripetal* Force, is that whereby any Body endeavours by Gravity to reach ſome Point, as its proper Center, the contrary, is called *the Centrifugal Force*, receeding from the Center.

In the next Place 'twill be neceſſary for the young Trigonometer to underſtand the *Laws of Nature* relating to *Motion*, or Bodies freely moving or falling; which Sir *Iſaac Newton* has reduced to Three, in his *Principia*; and are as follows.

Law I.

Every Body will continue in a State of Reſt, or will move uniformly in a right Line, except ſo far as it is compelled to change its firſt State by Forces impreſſed.

Law II.

The Change of Motion is always proportionable to the moving Forces impreſſed, and is always made according to the Right Line, in which that Force is impreſſed.

Law III.

Action is always equal and contrary to *Re-action*; that is, the Actions of two Bodies on each other are always equal, and in contrary Directions.

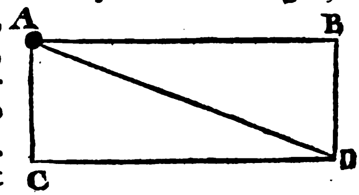
Theſe Laws of *Motion*, which all Bodies obſerve, are illuſtrated, proved and eſtabliſhed, by the above-named great Author in the ſaid Book, and by many other Writers of *mechanical* and *experimental Philoſophy*;

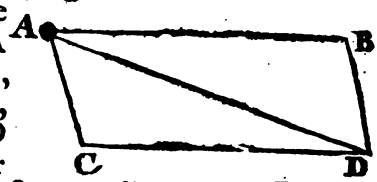
losophy; which the Reader may consult at his Pleasure.

From these *Axioms* or *Fundamental Laws of Nature*, the following *Corollary* is deduced.

Corollary.

Every Body by conjoint Forces, will describe the *Diagonal* of a *Parallelogram* in the same Time, wherein it would do the *Sides* by the *Forces* singlely.





Suppose a Body A, with the Force $M = 8$, were carried with an uniform Motion from A to B in the Time $T = 5$. and in the same Moment it were urged by the Force $N = 3$, from A towards C; it would, by those joint Forces, be borne from A to D in the same Time. For because the Force N acts according to the Line AC parallel to BD, it will nothing alter the Velocity of approaching to the Line BD produced by the first Force M, by *Law* II, it will therefore arrive to the Line BD in the same Line T, whether the Force N acteth on it or not, and will in the End of that Time be found somewhere in the Line BD. By the like Reasoning, at the End of the said Time it will be found somewhere in the Line CD; and therefore of Necessity it will be in the Point where these two Lines meet, *viz.* in D, and the Force (thus compounded of the first two M and N) whereby 'tis carried in the Line AD, will be $= \sqrt{\overline{M + N}}$, or $\sqrt{89}$ $= 9.8$. For since the Time is the same, the two Forces M

M, N, will be repreſented by the Sides AB = CD, and AC = BD, in the *firſt Caſe of a Rectangular Parallelogram*; but $\sqrt{\overline{AB + BD}} = \sqrt{\overline{AD}} = \sqrt{9.8}$ the compound Force.

In the *Second Caſe*, where the Direction of the Forces are impreſſed obliquely to each other, on the Body A; it will always hold, as the Sine of the greateſt Acute Angle : is to the greater Force (or as the Sine of the leſſer Acute Angle : to the leſſer Force) : : ſo the Sine of the Obtuſe Angle : to the new compound Force ſought. From hence reſults the Invention of that curious Art.

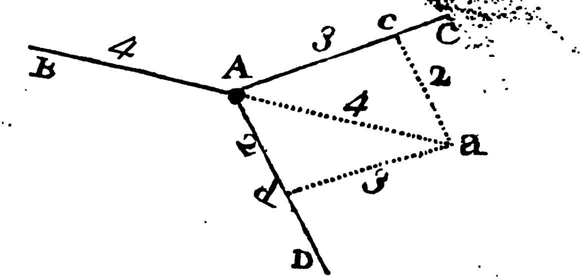
Of the Compoſition and Reſolution of Forces *and* Motion.

THE *Example* in the preceding *Corollary*, is ſufficient to ſhew, how two direct or oblique Forces AB and AC, are to be reduced to one direct Force AD, and how the Quantity of that Force is to be eſtimated.

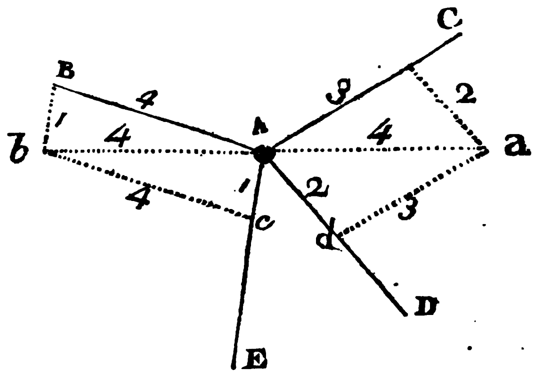
And if any ſingle direct Force AD, of a given Quantity, be propoſed; 'tis eaſy to ſee how it may be reſolved into two Forces AB and AC, that ſhall act either in right or oblique Directions to each other; and to determine the Quantity of each.

Alſo, if a Body A be acted upon by three given Powers or Forces B, C, D, that are as the Sides of a Triangle made by Lines parallel to the Directions of the Powers, 'tis from hence evident, that Body will be at Reſt.

Suppoſe



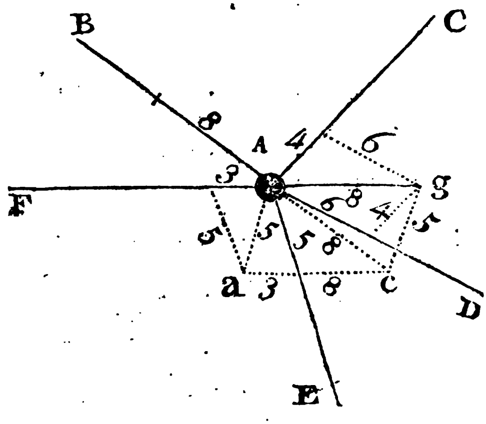
Suppose the Force B = 4, C = 3, and D = 2; then because the Forces at B, C, and D are as the Sides of the Triangle Aca, or Ada, 'tis plain the Forces at D and C act but as one at a; and because that Force is equal and contrary to the Force at B, the Body A must be at Rest. Thus 'tis easy to understand how a Body A may be at Rest, tho' acted upon by four different Powers at once in as many different Directions.



For suppose the Body A were sollicited towards E, D, C, and B, with Forces at E = 1, at D = 2, at C =

C = 3, and B = 4. Having compleated the Parallelograms, 'tis manifest the two Powers at D and C are reduced to one at a; and acts in the Direction Aa; also that the Powers at E and B, will act on A jointly in the Direction Ab; but the Powers at a and b are equal and contrary, and therefore the Body A must be at Rest.

Once more, a Body A, tho' acted upon by five different Powers, B, C, D, E, and F, may yet remain in Equilibrio, or be at Rest.

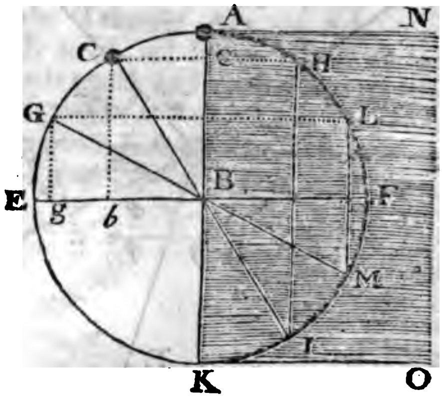


Let the Forces be as follows, at B = 8, at C = 4, D = 6, E = 5, and at F = 3; then having drawn the respective Parallelograms, it appears, that the Powers at C, D, are reduced to the Direction A g; the Powers at E, F, to the Direction A a; also those two Ag, and Aa, are reduced to the Direction Ac; but this is equal, and contrary to the Power and its Direction at B, and therefore the Body A must be at Rest.

And after this manner, it may be shewn how the Body A may abide at Rest, tho' attracted by any greater Number of Forces, and in as many Directions.

And now if we ſuppoſe the Powers or Forces ſuch as ſhall determine the Body A, to move in ſome Direction; if the Powers, and the Directions in which they act on the Body A be given, the Quantity of the Motion of the ſaid Body, and its Direction, may be eaſily found by Trigonometrical Calculation, as the Reader may try, and afterwards prove it by Experiments.

The Power, or Forces of Oblique Percuſſion compar'd.



'TIS a plain Axiom, that if any Body A be impelled with any given Force, in a perpendicular Direction on the Point B, that it will then ſtrike the ſaid Point B with the greateſt Force poſſible; which ſuppoſe of 1000 Parts; then from hence we ſhall eaſily compute the Force of the Stroke in any other Direction, obliquely, as in CB for Inſtance: For let the Force CB be reſolved into two other Forces, *viz.* Cc and cB, which together are equiva-

lent to the Force CB. But the Force Cc, being parallel to the Plane EF in which is the Point B, can by no means affect it, and therefore not to be considered. The Forec cB then, as being perpendicular, can only avail to strike the Point B; now cB = Cb, the Sine of the Angle of Inclination CBE; therefore the Force of any oblique Percussion is to a Perpendicular Stroke (*cæteris paribus*) as the Sine of the Obliquity or Inclination is to Radius; and the Force of one oblique Stroke CB is to the Force of any other oblique Stroke GB, as the Sine Cb is to the Sine Gg. The Computation of the Quantity of the Force of each Stroke on the Point B, supposing that of AB = 1000, here follows. Let the Angle CBE = 58° 30′, and the Angle GBE = 25°.

Then say, as Radius ——— ———		10.0000000
Is to the Force of the Perpen. Stroke ——	AB = 1000 =	3.0000000
So is the Sine of the Angle ——	CBE = 58° 30′ =	9.9353204
To the Force of the Stroke in the Oblique Direction	CB = 861.7 =	2.9353204

Then comparing Oblique Forces, say;

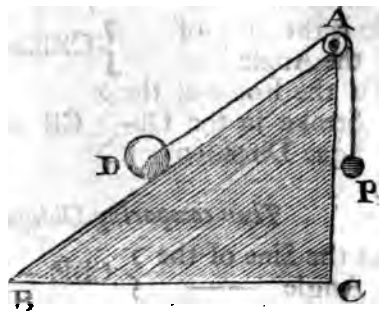
As the Sine of the Angle ——	CBE = 58° 30′ =	9.9353204
Is to the Sine of the Angle ——	GBE = 25° 00′ =	9.6259483
So is the Force of the Stroke in the Direction ——	CB = 861.7 =	2.9353204
To the Force of the Stroke in the Direction ——	GB = 422.6 =	2.6259483

Whence the Forces of the Strokes on the Point B, in the Directions AB, CB, and GB, are in the Proportion of the Numbers 1000, 861.7, and 422.6; or nearly as the Numbers 10, 8, 4.

And such will be the Proportion of a Quantity of Wind or Water ANOK striking against any Plain in a Right Position, as AK, and the Oblique Positions CI, or GM, as is evident by the bare Inspection of the foregoing Scheme; that is, the Sides AK, HI, LM, are as 10, 8, 4, *fere.*

Of the Descent of heavy Bodies on inclined Planes.

Case I.

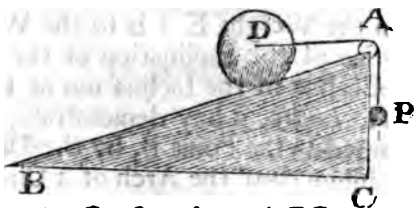


THAT is an inclined Plane which makes an Oblique Angle with the Horizon. As the Plane AB, which is inclined to the Horizon BC, in the Oblique Angle ABC; the Length of the Plane is AB, and its Height AC, and its Base BC; which together constitute a Figure of a Right angled Triangle; whence the Calculation of the Motions and Powers of Bodies descending, or abiding an inclined Plane, becomes the immediate Province of the Plain Trigonometer.

I. By the Writers of Mechanics, 'tis demonstrated, That if any Weight or Body D, be sustained on an inclined

inclined Plane AB, by any Power P, so that it remains Aquilibrio; the Power P : is to the Weight of the Body D, : : as the Sine of the Angle of Inclination AC : is to the Radius AB, in Case the Direction AD be parallel to the Plane.

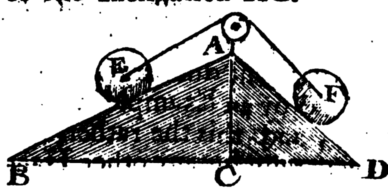
Case II.



But if the Direction AD be parallel to the Horizon BC, the Proportion will be, as the Power P : to Weight D, : : the Sine of the Inclination AC, : to the Co-sine thereof BC; or : : the Tangent : to the Radius.

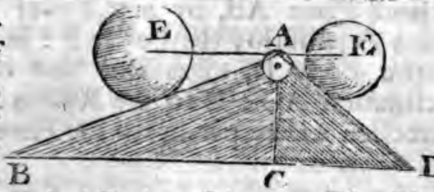
II. 'Tis demonstrated, That the Celerity of a Body rolling down an inclined Plane : is to the Celerity of a Body falling perpendicularly, in the same Time, : : as the Sine of the Inclination AC : is to the Radius AB.

III. 'Tis demonstrated, That the Time wherein the inclined Plane AB is passed over : is to the Time in which the Perpendicular is run thro' : : as the Radius AB : to the Sine of the Inclination AC.



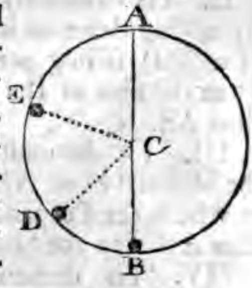
IV. 'Tis demonstrated, That if two Weights E, F, hold each other in Aquilibrio, on two inclined Planes AB and AD, by a Line whose Direction is parallel to the respective Planes; that then the Weight E : is to the Weight F, : : as the Sine of the Inclination of the Plane ADC : is to the Sine of the Inclination of the Plane ABC.

V.



V. 'Tis demonstrated, That if two Weights E, F, hold each other in Aquilibrio, on two inclined Planes AB and AD, by a Line in a Direction parallel to the Bases BD of the inclined Planes; that then the Weight E : is to the Weight F : : as the Co-sine of the Inclination of the Plane ABC : is to the Co-sine of the Inclination of the Plane ADC.

VI. *Lastly*, They demonstrate, That the Celerity acquired in the Point B, by the Descent of a Body or Pendulum thro' the Arch of a Circle DB, : is to the Celerity acquired in B, by its descent thro' any other Arch EB, : : as the Sine of $\frac{1}{2}$ DB : is to the Sine of $\frac{1}{2}$ EB.



These six foregoing and useful Theorems may be found demonstrated by all the Writers of mechanical Philosophy, as Dr. *Keil*, Dr. *Gravesande*, *Desaguliers* and *Hawkbee*, Mr. *Watts*, *Motte*, *&c.* which would have been quite besides my Purpose to have transcribed here; the Part of Calculation only is incumbent upon me, and that shall discharge, by giving an Instance of the Method by an Example in each of the foregoing *Theorems*; and this the rather, as I have not seen it done so generally by any other Hand.

Theorem

Theorem I. *exemplified*, Case I.

I. *What Power* P *is sufficient to sustain a Body* D *weighing* 112 *Pounds, on a Plane* AB *inclined to the Horizon* BC, *in an Angle* ABC = 37° 30′ ? say;

As the Radius — — —		10.0000000
Is to the Sine of the Inclination —	ABC = 37° 30′ =	9.7844471
So is the Weight of the Body —	D = 112 =	2.0492180
To the Power to be applied at —	P = 68$\frac{1}{10}$ =	1.8336651

Therefore 69 Pound is sufficient to raise the Body D = 112 Pounds on a Plane of that Inclination.

II. *What Weight* D, *may be sustained by a Power* P = 99 *Pounds, on a Plane whose Inclination* ABC = 81° 53′ ?

As the Sine of the Inclination —	ABC = 81° 53′ =	9.9956276
Is to Radius — — —		10.0000000
So is the Power applied at —	P = 99 =	1.9956352
To the Weight that will be sustained	D = 100 =	2.0000076

So that when the Inclination is so large, a Weight of 100 lb. will require above 99 lb. to move it.

III. *Suppose the Power* P $=$ $1\frac{1}{2}$ *Pound, sustains a Weight* D $= 5$ *Hundred Weight, or* 560 *Pounds, on an inclined Plane, what is the Inclination of that Plane?* Say thus;

As the Power applied at — P $= 1.5 =$ 0.1760913
Is to the Weight it sustains — D $= 560 =$ 2.7481880
So is Radius — — — 10.0000000

To the Plane's Inclination — } $=$ ABC $= 2° \; 8' =$ 8.5720968

Hence a Plane may have an Inclination so very small, that the smallest Power may sustain or move the greatest Weight thereon.

Theorem I. Case II. *exemplified.*

Let the first Question in the first Case be here resolved, in order to see the Difference. Therefore say;

As Radius — — — 10.0000000
Is to the Tangent of the Inclination } ABC $= 37° \; 30' =$ 9.8849805
So is the Body or Weight — D $= 112 =$ 2.0492180

To the Power that will sustain it — — } P $= 85\frac{9}{10} =$ 1.9341985

So that in this Case there is required $17\frac{8}{10}$ Pounds more to move the Weight D than in the former Case.

II. *To*

II. *To see the Difference between Case* I, *and* II, *in the second Question,* say;

As the Tangent of the Inclination } ABC = 81° 53′ = 10.8458261
Is to Radius —— —— —— 10.0000000
So is the Power applied at — P = 99 = 1.9956352

To the Body or Weight at D = $14\frac{1}{10}$ = 1.1498091

Hence it appears, that the Power P will not sustain one seventh Part of its own Weight on that Plane; nor will it support its own Weight if the Inclination be above 45°. And therefore a Plane in this second Case, can be useful only in an Inclination under 45 Degrees.

III. There is little or no Difference in the Result of the third Question, by this or the former Case; because the Sines and Tangents of very small Arches are nearly equal. When it appears that an inclined Plane is abundantly more useful in the first Case than in the Second.

Theorem II. *exemplified.*

Suppose a Body falling perpendicularly in 5 Seconds of Time, acquire 100 *Degrees of Velocity, how many Degrees of Velocity will it acquire in the same Time by descending on an inclined Plane, whose Angle of Inclination is* 30 *Degrees?* say;

As Radius ——— ——— ———		10.0000000
Is to the Sine of the Inclination	30° =	9.6989700
So the Velocity in the Perpendicular ——— ———	100 =	2.0000000
To the Velocity on the given Plane ——— ——— ———	50 =	1.6989700

That is just half the Velocity in the Perpendicular.

Thus, if the Velocity on the Plane, and the Inclination be given, the Velocity in the Perpendicular may be found.

Also, if both the Velocities are given, the Inclination of the Plane is thus easily found.

Theorem III. *exemplified.*

Suppose a Body descend thro' an inclined Plane, whose Inclination is 35° 50', *in* 16 *Seconds of Time, in how many Seconds will it fall from the Perpendicular Height of that Plane?* Say thus;

As Radius ——— ——— ——— ———		10.0000000
Is to the Sine of the Inclination ——— ———	35° 50' =	9.7674746
So is the Time of the Descent in the Plane ———	16° =	1.2041200
To the Time of Descent in the Perpendicular —	9" $\frac{1}{10}$ =	0.9715946

And thus may the two other Cases of this Theorem be solved.

Theorem

Theorem IV. *exemplified.*

Suppose the Inclinations of the two contiguous Planes AB *and* AD, *were* ABC $= 32° 14'$; *and* ABC $= 43° 25'$; *and it were required to find the Weight of the Body* F, *which on the Plane* AD *shall sustain or move the Body* E *on the Plane* AB, *weighing* 465 *Pounds?* Say.

As the Sine of the Inclination ———	ABC $= 43° 25'$	$=$	9.8371456
Is to the Sine of the Inclination ———	ABC $= 32° 14'$	$=$	9.7270272
So is the Weight of the Body	E $= 465$	$=$	2.6674529
To the Weight of the Body required ———	F $= 360$	$=$	2.5572345

And thus for any other of the Cases of this *Theorem.*

Theorem V. *exemplified.*

This is but as it were a second Case of the last; and therefore, to see the Difference between them, let the Example there be here again resolved. Saying,

As the Co-sine of the Inclination	BAC $= 57° 46'$	$=$	9.9273103
Is to the Co-sine of the Inclination	DAC $= 46.35$	$=$	9.8611608
So is the Weight of the Body	E $= 465$	$=$	2.6674529
To the Weight of the Body ——— ———	F $= 400$	$=$	2.6013034

Whence it appears, that in this Caſe 40 Pounds more is neceſſary, than in the Caſe of the laſt *Theorem*, to ſuſpend the Body E in Equilibrio, or to move it. And therefore the Pulley A ought ſo to be placed, both in the ſingle and double inclined Planes, that the Line coming from the Body over it may be parallel to the Plane.

Theorem VI. *exemplified.*

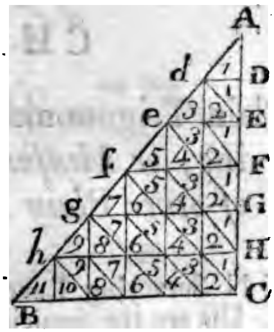
Suppoſe the Arch DB = 40°, *and the Arch* EB = 100°; *and that the Pendulum deſcends from* D *to* B, *and thereby does acquire* 15 *Degrees of Velocity in* B; quere, *how many Degrees of Velocity it will have in* B, *by falling thro' the Arch* EB? Say thus;

As the Sine of ——	$\frac{1}{2}$ DB = 20° =	9.5340517
Is to the Sine of ——	$\frac{1}{2}$ EB = 50° =	9.8842540
So are the Degrees of Velocity acquired by deſcending thro' ——	DB = 15 =	1.1760931
To the Degrees of Velocity acquired by deſcending thro' ——	EB = $33\frac{6}{10}$ =	1.5262954

That is, it will now be above twice as ſwift in B, as before.

To

To represent the Times, Velocities, and Spaces passed thro' by falling Bodies, by the Parts of Right-angled Triangle.



LET A be the Time in which a Body begins to fall, and Let AD be the first Minute, AE the second Minute, AF the third Minute, &c. Then shall the Side Dd be the Velocity acquired at the End of the first Minute, Ee the Velocity at the End of the second Minute, Ff the Velocity at the End of the third Minute, &c. *Lastly*, The Triangular Area ADd, will represent the Space passed thro' in the first Minute, AEe the Space passed thro' in the second Minute, AFf the Space passed thro' in the third Minute, &c. And thus if AC be six Minutes, the Side CB will be the Velocity, and the Area ACB the Space passed over at the End of the six Minutes.

From hence 'tis evident, that the Velocity of a falling Body is always proportionate to the Time; for as the Time AD : is to the Time AC : : so is the Velocity or Celerity Dd : to the Celerity CB; and therefore the Motion of falling Bodies is a Motion equally accelerated, or equally increased in equal Times.

Hence also the Spaces gone thro' from the Beginning of the Fall are as the Squares of the Times; for the Area ADd, the Space gone thro' in one Minute, : is to the Area ACB, the Space gone thro' at the End of six Minutes : : As the Square of the Time AD : is to the Square of the Time AC; Hence the

the Spaces increase according to the odd Numbers, 1, 3, 5, 7, 9, 11, &c. All this is evident from the Nature of a Triangle itself.

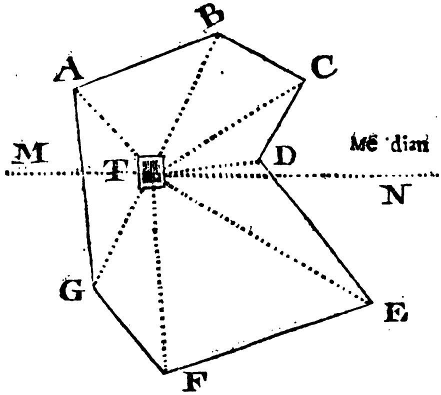
CHAP. VII.

Plain Trigonometry *applied to* Surveying, *or Measuring of Land; and also of other Plain Superficies.*

IN Surveying Land, the Instruments in common Use are the Semicircle, Plain Table, and Theodolite for taking the Angles, and a Protractor, or Plain Scale, and Compasses for Plotting or Delineating the Dimension of the Field on Paper, in order to reduce it to Triangles, and thereby to find the true superficial Content or Area in Acres, Roods, and Poles.

I shall here exhibit some of the best and most usual Methods for taking the Plot of a Field, and then shew how to find its Area; which may be as sufficient for the ingenious young Artist, as some larger Tract wherein abundance is often said to little Purpose; since a Word to the Wise is enough, and there is no making a Silken Purse of a Sow's Ear when a Person has said all he can.

I. *To take the Plot of a Field at one Station in any Part thereof, whence all the Angles may be seen.*



IMAGINE ABCDEFG, to be the Field whose Plot you are to take. Having order'd Marks to be set up at each Angle, and chosen a proper Place at T for your Station, there set up your Plane Table (for that is the most usual Instrument and answers the Purpose of a Semicircle or Theodolite) and turn it about till the Needle hang over the Meridian Line of North and South in the Chart of the Compass, there screw it fast on the Ball and Socket. Then direct your Sight to the Angle A, and observe what Degree is cut by the Index on the graduatd Edge of the Table, which you'll find to be 45 15. Then measure the Distance from T to A, which is five Chains and 90 Links; which note down in your Field-Book; then proceed doing the like by all the rest, until you have found the Quantity of the Angles, and Distances from your Station T to each Angle respectively, your Work being finished will stand as follows.

The

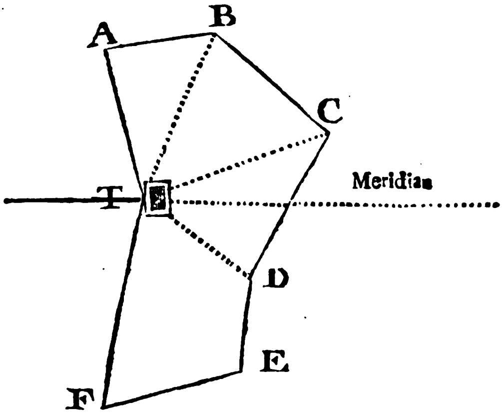
The Quantity of the Angles are

Deg.	Min.	Ang.
45°	15′	=A
115	15	=B
150	15	=C
178	15	=D
213	30	=E
268	30	=F
302	15	=G

The Quantity of the Distances from T to the Angles.

	Chains	Links
A	= 5	90
B	= 7	80
C	= 9	40
D	= 5	50
E	=13	50
F	=10	70
G	= 6	60

II. *To take the Dimension of a Field, at one Station, in any one Angle thereof, whence the others may be seen.*

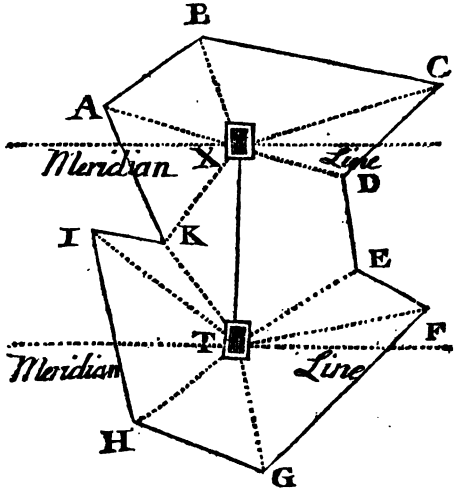


SUPPOSE ABCDEFT, the Field whose Plot and Dimensions you are to take; then having chosen your Station at the Angle T, whence you have a plain View of the rest, there set up your Table, adjusting it to the Meridian, and directing your Sight to A, observe the Degrees cut by the Index on the graduated Border, which are 76°; then measure the Distance from T to the Angle A, and you'll find it to be eight Chains; do the like by all the other Angles

Angles and Diſtances, and entering the Dimenſions of thus taken, the Work in your Field-Book will ſtand thus.

		Deg.	Min.			Chains	Links
The Quantity of the Angles ſtand thus.	A=	76	00	The Diſtances from the Table T to the Angles.	A=	8	00
	B=	114	00		B=	9	50
	C=	163	30		C=	10	40
	D=	220	30		D=	7	00
	E=	245	00		E=	10	20
	F=	284	30		F=	10	80

III. *To take the Plot and Dimenſions of an irregular Field at two Stations, whence all the Angles may be ſeen.*



LET the Field be the Figure ABCDEFGHIK, in which having choſe your two Stations at T and X, and placed Marks in all the Angles; then in T ſet up your Plain Table, and adjuſt the Needle

 to

to the Meridian Line; and direct the Sights to all the visible Angles EFGHIK, note down the Quantities of each, and also the Distances thereto, in your Field-Book. This being done, direct your Sights to the second Station at X, and note in your Book, what Angle it makes with the Meridian, and with it set down the Distance between the two Stations T and X.

Then remove your Table to X, and there fix it, with the Needle over the Meridian Line of the Compass; here take the Quantity of the remaining Angles ABCD, and your Distances from X to each of them, and having noted them down in your Field-Book, your Work is finished, and will stand thus.

		Deg.	Min.			Chains	Links
The Angles at the first Station.	E=	145	00	The Distances from the Station T to the Angles	E=	8	30
	F=	167	00		F=	10	20
	G=	255	30		G=	7	00
	H=	320	30		H=	6	90
	I=	38	30		I=	9	70
	K=	54	00		K=	6	60

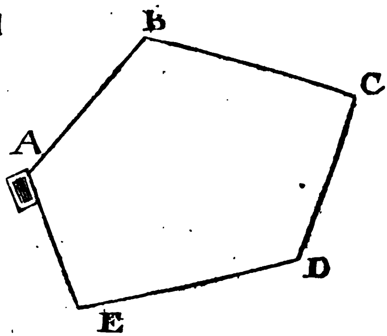
Here the second Station is at Right Angles to the Meridian of the first Station (and whatsoever the Declination of the second Station from the Meridian be, note it down.) The Distance of the two Stations TX is 10 Chains, and 50 Links; the Observations at the second Station stand thus.

		Deg.	Min.			Chains	Links
The Angles.	K=	309	00	The Distances from X to	K=	6	50
	A=	15	00		A=	7	50
	B=	69	00		B=	6	00
	C=	116	00		C=	11	20
	D=	196	00		D=	5	80

After this Manner you proceed with three or more Stations, when the extraordinary Largeness of the

the Plane make more than two Stations neceſſary, as that of Champain-Fields, large Heaths and Commons.

IV. *To take the Plot of a Field by going round the ſame.*

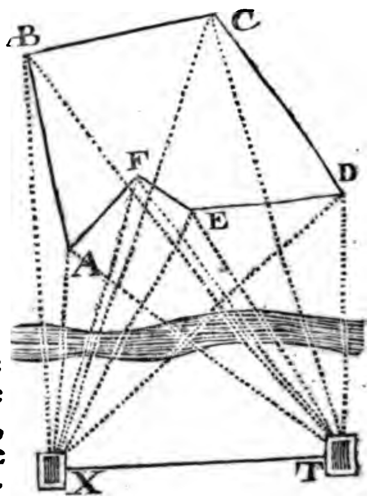


LET the Field be repreſented by the Scheme ABCDE. At the Angle A ſet up your Inſtrument, and laying the Index on the Diameter, turn it about till thro' the Sights you ſee the Angle E; there fix it faſt, and turn the Index about to the Angle B, and obſerve the Degrees cut thereby, which note in your Book; and then remove your Inſtrument to B, where do the like, and afterwards at the Angles C, D, and E; and as you go round meaſure the Sides AB, BC, CD, DE and EA. Which being all orderly ſet down in your Book, it will ſtand as follows.

		Deg.	Min.
The Angles	A=	120	30
	B=	113	00
	C=	87	00
	D=	122	30
	E=	97	00

		Chains	Links
The Sides	AB=	9	60
	BC=	11	20
	CD=	8	70
	DE=	11	80
	EA=	7	50

V. *To take the Plot of a Field, which you cannot approach to, at two Stations.*



LET ABCDEF be a Field situate on the other Side of a River from you, and consequently to which you cannot have Accefs, and yet you are required to take the Dimenfions thereof; in any fuch cafe proceed thus.

Chufe any two Stations, T and X, whence you can difcern all the Angles of the Field, and fet up your Table in T, and take (as before directed) the Quantity of the Angles fubtended by each Side of the Field, by directing the Sights to the Angles A, B, C, D, E, and F, fucceffively, and note them down in your Field-Book; this done, meafure with your Chain the Diftance to the other Station X; and there fet up and fix your Table, and take the Quantity of the Angles at this Station, as before you did at the other, and note them down in your Book alfo; and thus your Work being done, will ftand thus.

The

		Deg.	Min.			Deg.	Min.
The Angles from the first Station at T,	A =	40	15	The Angles from the second Station at X,	A =	82	30
	B =	55	00		B =	90	00
	C =	77	00		C =	68	30
	D =	93	00		D =	40	30
	E =	62	00		E =	58	00
	F =	56	00		F =	70	00

The Diſtance between the two Stations is 154 Chains.

VI. *To delineate on Paper the Obſervations taken according to any of the foregoing Methods, and thereby to form a Plot, or Map of the Field.*

IF you meaſure with the Plain Table, then a Field-Book is needleſs, becauſe you may very neatly fix a Sheet of white Paper on the Table, by means of the graduated Border, which you take off and put on again with the Paper under it, faſt fix'd to the Table; now on this Paper the Meridian Line is firſt to be drawn, and then by directing your Sights to each of the Angles, you draw the Lines TA, TB, TC, *&c.* (in the firſt Method, for Inſtance) indefinitely, with a Pencil; and on each of which you ſet off the Diſtance you meaſure from the Table thereto, as five Chains and 90 Links from T to A; then ſeven Chains and 80 Links from T to B, and join the Points AB, ſo you have plotted one Side of the Field; then ſet off 9 Chains and 40 Links from T to C, and join the Points BC, and thus you have plotted

plotted two Sides of your Field AB and BC; after the ſame Manner you proceed to plot off all the other Sides CD, DE, EF, FG, and GA; and thus is the Plot of the whole Field delineated before you go out of it, on the Table itſelf.

In this Manner a Plot of the Field is delineated on the Plain Table, according to the ſecond Method; and alſo according to the third Method, by carefully laying off the Diſtance of the Stations TX, in the due Angle of its Declination from the Meridian; thus alſo you plot the Sides of the Field in the fourth Method as you go round it. But in the fifth Method there being no Diſtance meaſured, you muſt draw the Lines TA, TB, TC, *&c.* to a ſufficient Length, at the firſt Station T; and then at the other Station draw the Lines XA, XB, XC, *&c.* till they interſect the former Lines in the Points A, B, C, *&c.* then joining thoſe Points of Interſection, you form the Sides AB, BC, CD, *&c.* and conſequently finiſh the Plot of the Field.

If you ſurvey with another Inſtrument than the Plain Table, as the Theodolite, Circumferentor, or Semicircle; you muſt then have your Field-Book, wherein to ſet down the Dimenſions of Sides and Angles, as in the Examples of each of the foregoing Methods.

Then in order to delineate the Plot of the Field, according to thoſe Obſervations, you muſt take a clean Sheet of Paper, and on it draw the Meridian Line, as MN, in the firſt Method. Then take a Protractor, and fix the Center on the Point T, and its Diameter on the Meridian Line MN; then by the graduated Limb of the Protractor, make a Point at each of the Degrees and Minutes marked in your Book, as at 45° 15′, 115° 15′, 150° 15′, *&c.* and thus ſhall the Points A, B, C, *&c.* be deſigned on the

the Paper; having laid off the Quantity of all the Angles under 180°, you may either turn the Protractor, and mark out the Points for the Angles between 180° and 360°; or you may let the Protractor bide in its first Position, and draw blank Lines below the Diameter, according to the Quantity of the Angles in your Book. Having done this, remove the Protractor, and thro' the Points A, B, C, &c. draw the Lines TA, TB, TC, &c. and on the said Lines lay off the Lengths in Chains and Links, as in the Table, *viz.* 5 90 from T to A, 7 80 from T to B, &c. on each of the other. *Lastly*, Draw the Lines AB, BC, CD, &c. and thus the Plot or Map of the whole Field is compleated according to the first Method.

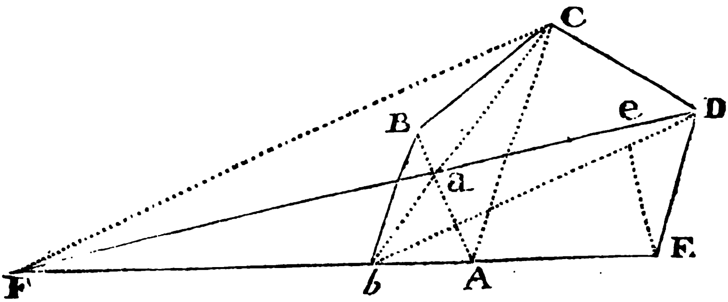
And any one who understands the Practice of Plotting of a Field by the Protractor, in the Manner taught in the first Method, will easily do it in any of the other; especially if what is said in Plotting by the Plain Table be well considered.

N. B. That if your Plain Table have not the Compass, or not a good one to be depended on, you cannot make use of a Meridian Line; but instead of that you must lay the Index on the Diameter of the Table, and then turn the Table about till thro' the Sights you 'spy the Angle A, there fix it fast; and then direct your Sights to the other Angles severally, and measuring to each the Distance from the Station T, you may plot it off as before taught. And indeed this is the most infallible Way, and in which you cannot be well mistaken, unless by any Errors in the Quantity of the Angles and Distances, which ought to be very carefully measured.

VII. To

VII. *To reduce a Multangular to the Form of a Triangle of equal Area.*

TO do this readily in Practice, you ought to be provided with a good Parallel Ruler; or in Want of that, it may be expeditiously done by the Geometric. *Prob.* VI. The Method of performing this most useful Proposition is as follows.



Let ABCDE be the Multangular Figure to be reduced; then, having continued the Side EA to F, lay one Side of the Parallel Ruler to the Points AC, and opening it to the Point B, draw Bb parallel to AC, and cutting EF in b. This being done, lay one Side of the Ruler to the Points bD, and opening it to the Point C, draw CF parallel to bD, and cutting EA produced in F, join FD; then shall the Triangle EFD, be equal to the first Multangular Figure ABCDE.

For the Triangle baA = aBC, therefore the quadrilateral Figure bCDE = ABCDE; and because baE = aCD; therefore the Triangle EFD = bCDE = ABCDE.

The

The Reaſon and Demonſtration of all this is evident by Inſpection of the Scheme to any one who underſtands the Geometrical *Theorems* IXth and Xth at the Beginning of the Book.

VIII. *To find the Area, or Superficial Content of a Field, in Acres, Roods and Perches, or Rods.*

THIS may be done two ſeveral Ways; *Firſt*, By reducing the Plot of the Field into a Triangle equal thereto, as taught in the laſt Propoſition. Or, *Secondly*, By dividing or reducing the Plot into ſeveral Triangles (when it cannot be conveniently reduced to one) by drawing Diagonal Lines from Angle to Angle.

Example of the firſt Method.

Suppoſe the Figure (in the Scheme to the laſt Prop.) ABCDE *were the Plot of a Field, whoſe Content was to be found. In the firſt Place I reduce it to the equal Triangle* EFD; *then by the ſame Plain Scale, by which the Plot was delineated, I meaſure the Baſe, or greateſt Side* DF, *and find it to be* 73 *Chains and* 50 *Links; and the Perpendicular Height* Ee, 13 *Chains and* 40 *Links.* Wherefore to find the Area,

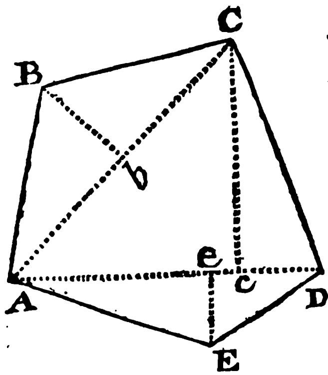
 Multiply

Multiply the Base 73 Ch. 50 Links=7350 Links.
By half the Height 6 Ch. 70 Links=670 Links.

	514500
	44100
The Product is Acres ——	49.24500
	4 Roods=1 Acre
Roods ——	0 980
	40 Perch =1 Rood
Perches ——	3.9200

	Acs.	Rds.	Per.
The Content then of the Field ABCDE is —	49 :	00 :	3

An Example of the second Method.



Let the Figure ABCDE be the Plot of a Field; draw the Diagonals AC and AD; then is the Plot reduced to the three Triangles ABC, ACD, and ADE; let fall the Perpendiculars Bb, Cc, and Ee; now AC×½Bb, or Bb×½AC =Area of the Triangle ABC. And $\frac{Ee \times Cc}{2} \times AD$, or $\overline{Ee \times Cc} \times \frac{1}{2}AD$=Area of both the Triangles ACD and ADE, to which add the Triangle before found ABC, and you will have the Area in Acres, Roods, and Perches of the whole Field ABCDE.

N. B. The Reason why in the foregoing Operation, there was cut off 5 Figures, to the Right-Hand, of the Product of Links, is because 100000 Square Links

Links is an Acre; and therefore 100000) 4924500 (=49.245 Acres, and Decimal Parts. To understand how 100000 Square Links is a Square Acre, you must observe, that every Chain is=4Rods=100 Links; and that 160 Square Rod=one Square Acre; divide therefore 160, the Square Rod in an Acre, by 16, the Square Rods in a Square Chain, the Quotient is 10, the Square Chain in an Acre. But every Square Chain=100x100=10000 Links, and 10000x10=100000 Links, as above.

From what I have delivered in this Chapter, 'tis easy to conceive how the Superficial Content or Area of any Right-lined Figure may be known; the Method being the same, the Dimensions only are to be considered as Rods, Yards, Feet, Inches, *&c.*

CHAP. VIII.

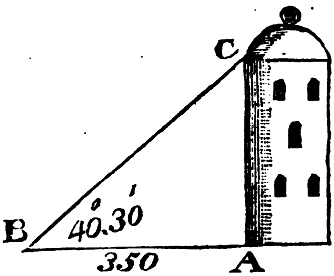
Plain Trigonometry *applied to* Altimetry, *and* Longimetry; *or the Measuring of Heights, Depths, and Distances, both Accessible and Inaccessible.*

A*Ltimetry* is the Art of measuring the Altitudes or Heights of Objects; whether they be Accessible, or such as we can approach unto, in order to measure from the Foot or Basis of the Objects; or Inaccessible, that is such as we can by no means come at, by reason of Water, or other Impediments. Also the Measuring of Depths is hereby understood and included.

I. *To measure the Altitudes of Objects Accessible.*

SUPPOSE the Height of the Tower AC, were required; in order to take it, you need only measure some convenient Distance from the Basis thereof, as from A to B, which suppose to be 350 Feet; then at B take with a Quadrant, or other graduated Instrument, the Angle ABC, subtended by the Height AC. Hence you have given the Side or Base AB, and the Angles, in the Right-angled Triangle ABC, to find the Perpendicular, or Height of the Tower AC.

Suppose then the Angle at B *were found by Observation to be* 40° 30′. *Then by* Case I. *of Right Triangles,* say;



As Radius —— —— —— —— 10.0000000
Is to the Side or Distance AB = 350 = 2.5440680
So the Tangent of the Angle —— —— } ABC = 40° 30′ = 9.9314989

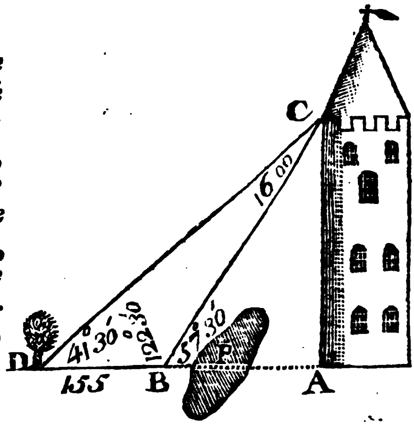
To the Perpend. or Height of the Tower, as required } AC = 298.9 = 2.4755669

Thus also you may find the Distance of your Eye from the Top of the Tower, or the Visual Line BC.

II. *To*

II. *To measure the Altitudes of Objects inaccessible, which stand on the Horizon.*

Suppose AC *to be the Object, whose Altitude or Height you would know, but cannot approach near it, to measure from the Foot of it, by reason of a large Lake or Pond* P *intercepting all Access.*



IN such Cases, proceed thus; measure from D to B, which suppose you find to be 155 Feet. Then at B take the Quantity of the Angle ABC, suppose 57° 30′; also take the Angle ADC, suppose you find it 41° 30′. Then you will have given, in the Oblique Triangle BDC, the Side BD, and the Angles; and therefore by *Case* I. of Oblique Triangles, you will find the Side BC to be 372.8 Feet, thus;

As the Sine of the Angle	BCD=16°00′=	9.4403381
Is to the Side —— ——	BD=155=	2.1903317
So is the Sine of the Angle	BDC=41°30′=	9.8212646
To the Side —— ——	BC=372.8=	2.5712582

Then

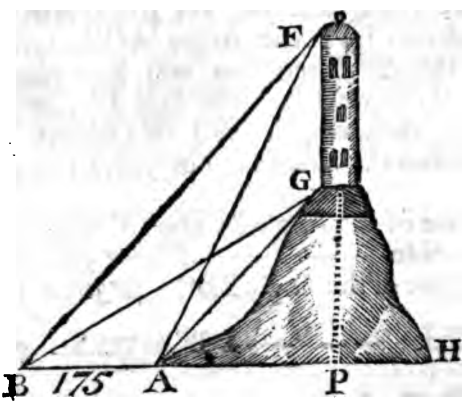
Then in the Right-angled Triangle ABC, there is known the Angles, and the Side or Hypothenuse BC; whence by *Case* III, there will be found the Perpendicular AC, or Height of the Tower thus;

As Radius — — — —		10.0000000
Is to the Side or Hypothenuse — —	BC = 372.8 =	2.5712582
So is the Sine of the Angle — —	ABC = 57° 30′ =	9.9260292
To the Altitude of the Tower — —	AC = 314.2 =	2.4972874

And now also the Distance BA may be found.

III. *To measure Altitudes of Objects inaccessible, and situated above the Horizon, as on a Hill, &c.*

Let FG *(in the following Figure) be the Inaccessible Object, situated on the Top of the Hill* AGH, *whose Height is required to be found at the Place* B; to find which proceed thus;



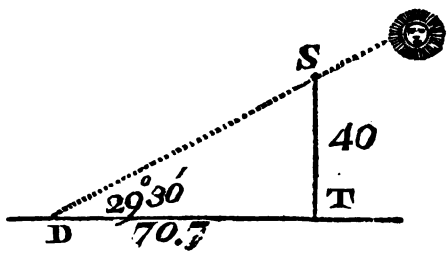
At

At B take the Angle ABF and ABG, and ſuppoſe you find the firſt 47° 45′, and the latter 30° 30′; then meaſure the Diſtance from B to A, which let be 175 Feet. *Laſtly*, At A let there be taken the Angle PAF=62° 30° ſuppoſe, and the Angle PAG=46° 30′. Then 'tis evident there are two Oblique Triangles ABG, and ABF by this means conſtituted; in each of which there is given all the Angles and the common Side BA.

From whence, in the firſt Triangle, there may be found the Side AF; as in the laſt Example. Then in the Right-angled Triangle PAF, there is known the Angles, and the Hypothenuſe AF, whence the Diſtance AP, and Height of both Hill and Tower together PF, may be found by *Caſe* III. Then in the Right-angled Triangle APG, there is known the Side AP, and all the Angles; whence by *Caſe* I. we can find the Height of the Hill PG; which ſubduct from the Altitude of both Tower and Hill (before found) PF, and there will remain GF, the Height of the Tower ſought. All this is ſo clear, as to make Examples, for every Step, unneceſſary.

IV. To

IV. *To take the Altitude of the Sun by the Shadow of a Staff.*



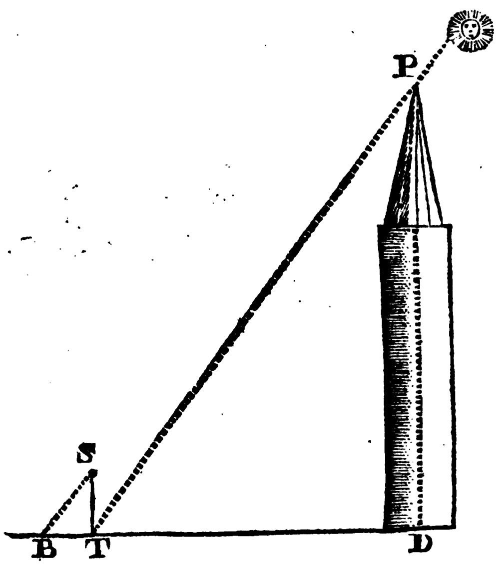
LET ST be a Staff of 40 Inches length, erected precisely on the Plain Superficies DT, the Sun shining thereon; observe how far the Shadow thereof is projected, suppose from T to D 70.7 Inches; then in the Right-angled Triangle DTS, (made by the Staff ST, the Shadow TD, and the Solar Ray SD,) there is known the Perpendicular ST, and the Base TD, to find the Angle at Base TDS, by *Case* IV, thus;

As the Bass, or Shadow, DT=70.7=	1.8494180
Is to the Height of the Staff TS=40=	1.6020600
So is Radius —— —— ——	10.0000000
To the Height of the Sun TDS=29° 30'=	9.7526420

And thus may the Sun's Altitude above the Horizon be found at any time very easy and readily.

V. *By knowing the Length of the Shadow of any Object, to determine its Height.*

LET P be the Pinacle of any Tower, or the Summit of any Object, and let its Shadow be projected to T, in the very Extremity of the Shadow T, ſet a Staff, as ST, truly upright, and obſerve the Length of its Shadow TB. Now the Triangles BST and TPD being ſimilar to each other, it will be BT : ST : : TD : DP. But the three firſt Terms are known, and therefore the fourth DP, the Height of the Object, will alſo be known.



Longimetry.

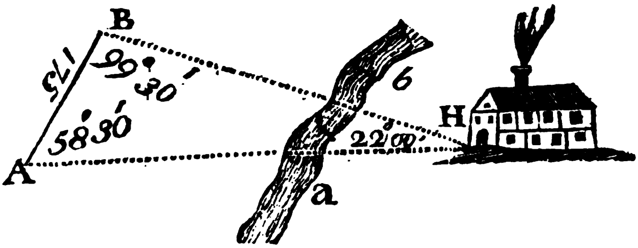
Longimetry is the Application of *Plain Trigonometry* to the Menſuration of the Diſtances of remote

 Objects

Objects; and those likewise are such; some of which admit of Access, and some will not; yet both Sorts are equally within the Reach or Limits of this excellent Art.

I. *To measure the Distance of any remote Object, Accessible or Inaccessible, at two Stations.*

Suppose a Person at A *were desirous to know the Distance of the House at* H *beyond the River* a b; the Method for finding it is thus;



At any convenient Distance from the Place A, let there be set up some Mark, as at B; then with a proper Instrument at A, find the Quantity of the Angle HAB = 58° 30′ suppose; then measure the Distance from A the first Station, to B the second, which let be 175 yards. *Lastly*, At B let the Angle ABH be taken, suppose 99° 30′. Then you have constituted the Oblique-angled Triangle HAB, wherein there is given all the Angles, and the Side AB, to find the Side AH, as also the Side BA; by *Case* I, of Oblique Triangles, thus;

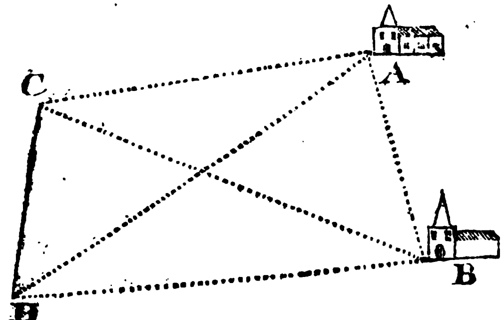
As

As the Sine of the Angle	AHB = 22° 00′ =	9.5735754
Is to the Side ——	—— AB = 175 =	2.2430380
So is the Sine of the Angle ——	ABH = 99° 30′ =	9.9940027
To the Distance ——	AH = 450.2 =	2.6534653
So also is the Sine of the Angle ——	BAH = 58° 30′ =	9.9307658
To the Distance ——	BH = 398.3 =	2.6002284

Hence it appears the House 450 Yards, or somewhat ¼ of a Mile from the first Station at A; and 398 Yards from the second Station at B.

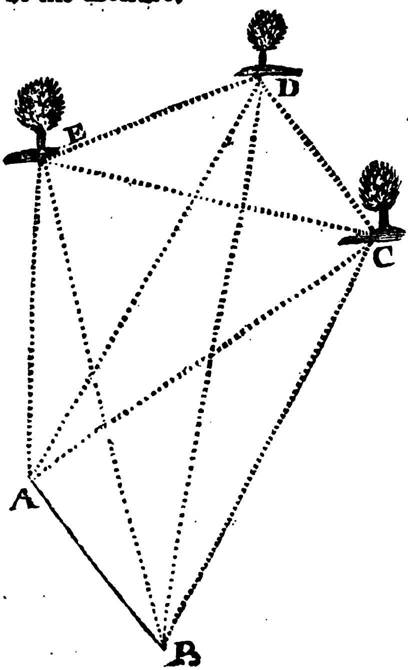
II. *To take the Distance between two, three, or more remote Objects, as Ships on the Sea, Towers or Spires in a City.*

Suppose it be required to find the Distance between the two remote Objects A *and* B; *and likewise how far they are from you at* C *or* D. Proceed in this Manner;



AT D let there be taken the Angles BDC= 76° 30′; and ADC=50° 30′. Then measure the Distance from your Station at D to some other, as C, which let be 200 Yards; then at C, take the Angles ACD=107° 30′, and BCD=77° 00′.

By this Means you have formed two Oblique Triangles, ADC and BDC, in each of which there are given all Angles, the common Side DC; when by *Case* I. there will be found the Sides CA and DA, the Distances of the Object A from the Stations D and C; likewise there may be found the Sides CB and DB, the Distances of the Object B from each Station. Then in the Triangle ACB or ADB, there are given two Sides, and the Angle included between them to find the third Side AB, the Distance of the Objects from each other, by *Case* III. of Oblique Triangles; and according to what is given the young Learner, may calculate the said Distances in Yards at his Leisure.



Again, suppose the Distance of three Objects, C, D, E, were required from each other, and from the two Stations AB. Now these may all be found as before; for by your Instruments, and measuring the Side AB, have obtain'd all the Angles and a Side AB in the Oblique Triangles ACB, ADB, AEB; whence the other Sides in each may be found as before; and therefore we shall have in the

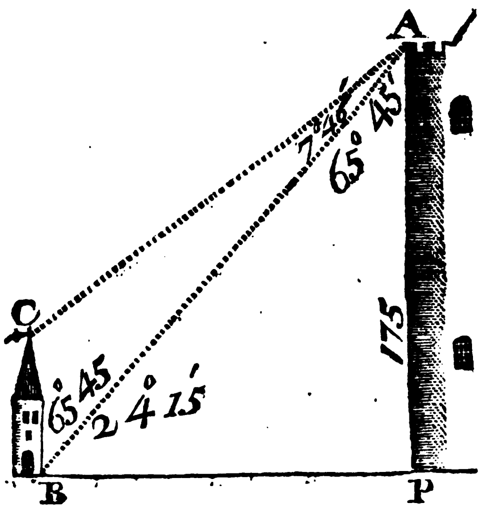
Oblique

Oblique Triangles EAC, DAC, EAD, two Sides, and an Angle included, given, to find the third Side in each, *viz.* EC, DC, and ED, the Distances of the Objects from each other respectively.

And thus may the Distances of many remote Objects be taken by a good Instrument in taking and plotting the Angles and Sides.

III. *Being on the Top of a House, Tower, or other high Object, to find from thence the Altitudes, Depths, and Distances of Objects on the Horizon.*

Suppose from the Top of the Tower PA, *whose Height is known, you behold at a Distance the Object* BC, *whose Distance* PB, *and Height* BC *you would.*



I. With

I. WITH a Semicircle take the Angle PAB = 65° 45′ suppose, and let the Altitude of the Tower PA be 175 Feet; then in the Right-angled Triangle ABP, there are given all the Angles and the Perpendicular PA, to find the Base BP and Hypothenuse AB, by *Case* II. thus;

As the Sine of the Angle ABP = 24° 15 =		9.6135446
Is to the Height of the Tower PA = 175 =		2.2430380
So is the Sine of the Angle BAP = 65° 45 =		9.9598815
To the Distance of the Object PB = 388.5 =		2.5893749
And so is Radius, to the Hypothenuse ——	BA = 426 =	2.6294834

II. Then for the Height BC, let the Angle PAC be taken, which suppose 73° 31′. Then in the Oblique-angled Triangle BAC, there is given the Side BA, and all the Angles (for the Angle ABC = BAP, because the Lines PA and BC are parallel; and the Angle BAC is the Difference of the Angles PAB and PAC, both known by Observation.) Therefore on the Gunter, say; as the Sine of 73° 31′ (the Complement of BCA = 106.° 29′ to 180) : Is to the Sine of BAC = 7° 46′ : : So is the Side BA = 426 : to the Side BC = 60 Feet, the Height of the Object which was required.

Thus if the Distance of any Object BC were first known, the Height or Altitude of any Object, on which you stand, as PA, may in the same Manner be found.

These are all the Principal Methods of finding the Altitudes, Distances, &c. of distant Objects; I know

know there are some other Methods of performing these Matters, such as they are, which I do not think worth while to trouble the Genuine Artist withal; the Deductions of his own Knowledge will readily supply him with all the Methods of Expedition, when Exactness is not required: And as the Purposes here treated of are very common, and of great Concern in Life, it evidently appears how wise and necessary an Art *Plain Trigonometry* is, by which they are all performed.

CHAP. IX.

Plain Trigonometry *applied to* Optics, *in its two Branches of* Catoptrics *and* Dioptrics; *shewing how to calculate the Course, Position, Focus, or Point of Convergence, &c. of Rays of Light after Reflection and Refraction, made by passing thro' different Mediums.*

OPTICS is that curious Science which treats of the Nature and Phænomena of Lights and Colours, and of the Nature and Manner of Sight or Vision; and how it is effected by the surprising Construction and Mechanism of the Eye. This Science is the Subject of no small Part of Natural Philosophy and the Mathematicks; it is divided into two principal Parts, *viz.*

Catoptrics, which treats of Light reflected from the Surfaces of Bodies, and explains the Laws and Properties

Properties of Reflection. This is very diverting as well as useful Part of Knowledge; and hereby those crafty Knaves whom the Ignorant call Cunning Men, or Conjurers, have been supplied with the most falacious Artifices to deceive the wondering, ignorant vulgar; who generally thought the surprising Phænomena of this Art were the Effects of Divination.

Dioptrics, which treats of refracted Light, or the Manner how a Ray of Light passes thro' different Mediums, as Air, Water, Glass, &c. and the Degrees of Refraction it suffers thereby. Here follows Sir *Isaac Newton's* Definitions and Axioms relating to the Particulars of this Science, as they stand in the first Pages of his excellent Treatise of *Optics*.

Definition I.

By the Rays of Light, says he, I understand is least Parts; and those as well successive in the same Line, as contemporary in several others.

Definition II.

Refrangibility of the Rays of Light, is their Disposition to be refracted or turned out of their Way, in passing out of one transparent Body, or Medium, into another; and a greater or less Refrangibility of Rays is their Disposition to be turned more or less out of their way in like Incidences on the same Medium.

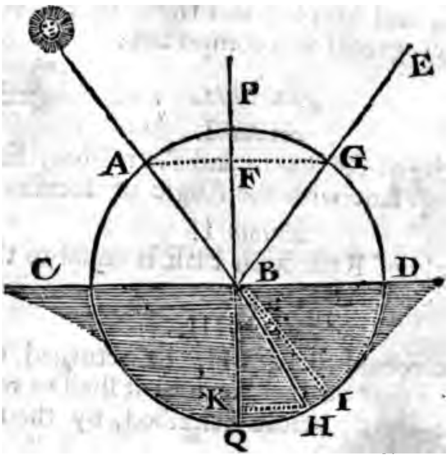
Definition III.

Reflexibility of Rays is their Disposition to be reflected or turned back into the same Medium, from any other Medium upon whose Surface they fall. And Rays are more or less reflexible, which are turned back more or less easily.

Defini-

Definition IV.

The Angle of Incidence, is that Angle which the Line deſcribed by the incident Ray AB, contains with the Perpendicular PB, to the reflecting or refracting Surface CD, at the Point of Incidence B, *viz.* the Angle ABP: See the following Scheme.



Definition V.

The Angle of Reflection (*viz.* PBE) or Refraction (*viz.* HBQ) is the Angle which the Line deſcribed by the reflected Ray BF, or refracted Ray BH, containeth with the Perpendicular PQ, to the reflecting or refracting Surface CD, at the Point of Incidence B.

Definition VI.

The Sines of Incidence AF, Reflection GF, and Refraction KH, are the Sines of the Angles of Incidence, Reflection, and Refraction.

Definition VII.

The Light whose Rays are all alike refrangible, I call simple, homogeneal, and similar; and that whose Rays are some more refrangible than others, I call compound, heterogeneal, and dissimilar.

Definition VIII.

The Colours of homogeneal Light, I call primary, homogeneal, and simple; and those of heterogeneal Lights, heterogeneal and compound.

Axioms.

Axiom I.

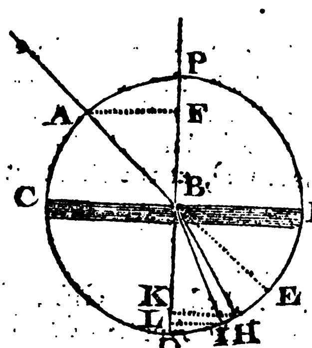
The Angles of Reflexion and Refraction, lie in one and the same Plane with the Angle of Incidence.

Axiom II.

The Angle of Reflection PBE is equal to the Angle of Incidence ABP.

Axiom III.

If the refracted Ray BH, be returned directly back to the Point of Incidence B, it shall be refracted into the Line AB, before described, by the incident Ray.



Axiom IV.

Refraction out of a rarer Medium into a Denser, is made towards the Perpendicular; that is, so that the Angle of Refraction is less than the Angle of Incidence.

Axiom V.

The Sine of Incidence is either accurately, or very nearly in a given Ratio, to the Sine of Refraction. Whence if that Proportion be known in any one Incidence, it is known in all others, for that Body and Medium.

It

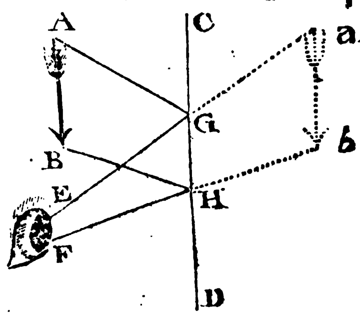
It is found by Experiments, that if the Refraction be made out of Air into Water, the Sine of Incidence is to Sine of Refraction, as 4 to 3. Wherefore if CD be the Surface of stagnating Water, and B the Point of Incidence in which any Ray coming in the Air from any Point A in the Line AC, it will be refracted out of its direct Course BE, into some other BH, making the Angle of Refraction HBQ, whose Sine KH : AF : : 3 : 4.

If CD be a Superficies of Glass, the Ratio of the Sine of Incidence, to the Sine of Refraction in Glass, is found by Experiments to be as 17 to 11; therefore the Incident Ray AC shall be refracted from its direct Course BE, to some other BI, constituting the Angle of Refraction IBQ, whose Sine LI : AF : : 11 : 17.

These Things premised, I shall proceed to the ensuing Propositions of *Catoptrics* and *Dioptrics*, which tho' they are common, may yet be somewhat serviceable to initiate the young Tyro in those kind of Speculations.

PROPOSITION I.

To exhibit the reflected Rays of Light from any Plane Surface; and the apparent or visible Place of the Object.



LET CD be the reflecting Surface, and AB the Object at a Distance therefrom; also let AG and BH, be two Rays of Light proceeding from the Extremities of the Object, and fall on the Points G and H: Now in order to find the Course of those Rays after Re-

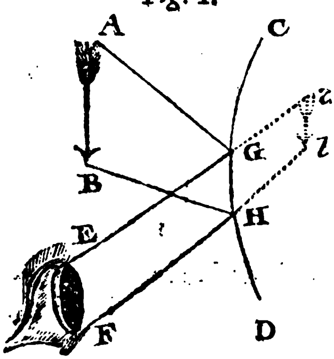
flection; make the Angle DGE=the Angle AGC; then shall the reflected Ray describe the Line GE, by *Axiom* II. After the same Manner it will be found, that the other Ray BH will be reflected in the Line HF; continue out EG to a, and FH to b, so that it may be Ga=GA, and Hb=HB: Then shall a b be the Spectrum or Image of the Object AB, in its apparent Place; which will be always equally distant, and similarly situated behind or beyond the reflecting Surface CD, as the Object is before it.

Suppose CD a common Looking-Glass; then from hence 'tis evident the Object AB being reflected in the Lines GEHF, to the Eye at EF, will have the same Effect on the Eye, as if the Object were really at a b behind the Glass, where it will therefore of Consequence appear to the Beholder. And thus the whole Theory of the Phænomena of a Looking-Glass, or Speculum is plain to be understood.

PROP. II.

To exhibit the Rays of Light reflected from spherical Surfaces, either Convex or Concave; and the apparent Place of the Object.

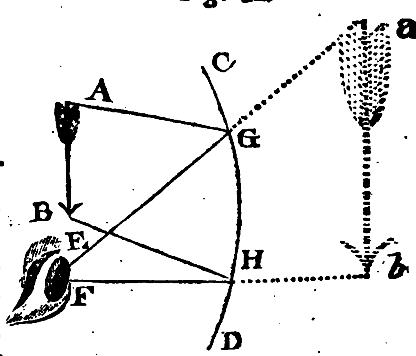
Fig. I.



SUPPOSE CD be any reflecting Surface, either Convex, as in *Fig.* I. or Concave, as in *Fig.* II. and let AB be the Object; and AG, BH, the Rays of Light coming from the Object and falling on the Surface CD. Let the Angles DHF, and DGE be made equal to the Angles CHB and CGA; then shall the reflected

reflected Rays describe the Lines GE and HF; continue those Lines beyond the Surface CD to a and b; and there will the Image of the Object be represented to the View. The different Properties of those three Sorts of Speculums, *viz.* Plane, Convex

Fig. II.

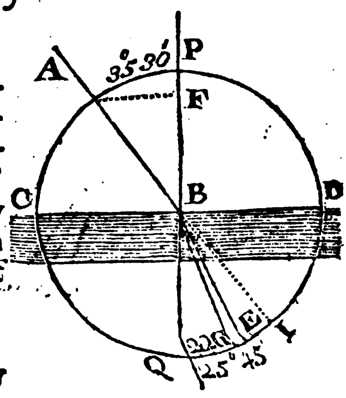


and Concave, are chiefly these which follow; the Plane Speculum presents the Thing in its due Situation, Distance, and Magnitude; the Convex or Gibbous Speculum exhibits the Object much nearer and much less than the Plane one; and *Lastly*, The Concave Speculum vastly encreases the Bulk, and pretty much the Distance of the Objects. There are abundance of other curious Phænomena of these Glasses to be met with in Books on this Subject.

PROP. III.

The Angle of Incidence being given, to find the Angles of Refraction from Air into Water or Glass.

SUPPOSE the Incident Ray AB be inclined to the Perpendicular PB in an Angle of 35° 30′, *Quere*, what Quantity the Angle of Refraction must be, in a Medium of Water?



By

By the Sliding Rule, say;

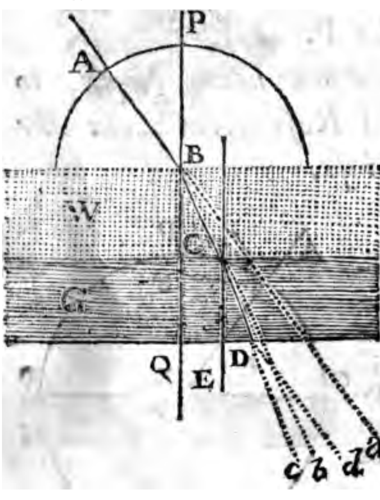
As 4 on the Line of Numbers is to 35° 30′ on the Line of Sines; so is 3 on the Line of Numbers to about 25° 45′ on the Line of Sines; in such an Incidence the Angle of Refraction in Water is thus found to be 25° 45′ = the Arch QE.

To find the Quantity of the same Angle in a Medium of Glass, say by the Rule;

As 17 on the Line of Numbers : 35° 30′ on the Line of Sines; : : So is 11 on the Line of Numbers : to 22° on the Line of Sines. The Angle therefore of Refraction will, in a Substance of Glass, be about 22° in such an Incidence.

PROP. IV.

The Angle of Incidence being given, to trace the refracted Ray thro' a Medium of Water, and then of Glass, contiguous to each other.



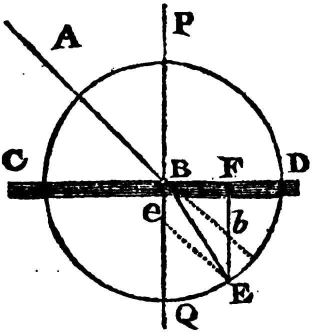
LET AB be a Ray of Light passing thro' the Air from A to B, whose Inclination to the Perpendicular PQ, is the Angle of Incidence ABP =35° 30′ suppose; at B it impinges on the Body or Medium of Water W, and, by the last *Prop.* it appeared, that a Ray of Light inclined to the Perpendicular in an Angle of 35°

$35° 30'$ in Air, it would be inclined thereto in paſſing a Medium of Water in an Angle of $25° 45'$, make therefore the Angle $CBQ=25° 45'$, 2nd BC ſhall be its Path thro' the Medium of Water. At C let it meet with the Surface of a Medium of Glaſs G; now by the laſt *Prop.* it appears alſo, that in the aforeſaid Inclination in Air, the Inclination in Glaſs will be $22°$; wherefore make $DCE=22°$, and CD ſhall be the Path of the Ray thro' the Medium of Glaſs Whence we obſerve, that the Ray AB is diverted from its direct Courſe in the Air, which tended to *a* in the Line Aa, and by the Medium of Water is refracted or bent into the Courſe Bb, in which Aqueous Path it would have perſiſted, had it not been interrupted by the Glaſſy Surface at C; by this more denſe Medium of Glaſs it became refringed, or again bent into the Direction Cc, in which it proceeds till meeting again with the Air at D, is refracted into the Line Dd parallel to its firſt Direction Aa.

N. B. The Refraction in Glaſs out of Water immediately may be thus diſcovered. Let the Sine of Incidence in the Air be called I, the Refraction in Water R, and the Refraction in Glaſs r: Then becauſe $I : R :: 4 : 3$, and $I : r :: 17 : 11$; it will be $3I = 4R$, and $11I = 17r$; conſequently it will be as $3 : 11 :: 4R : 17r$; therefore $44R = 51r$, whence $R : r :: 51 : 44$; that is, the Sine of Incidence : is to the Sine of Refraction, out of Water into Glaſs : : as 51 : to 44. Q. E. I.

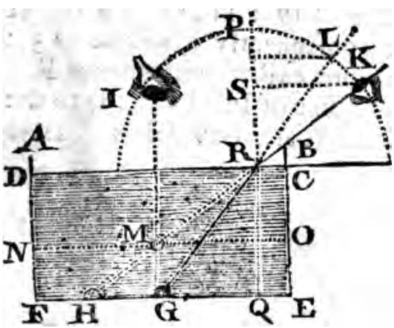
Prop.

PROP. V.



In the Scheme adjoined, let AB be the Incident Ray of Light, and BE the fame Ray refracted from its direct Courfe Ab, draw FE parallel to the Perpendicular PQ, and draw Ee parallel to Bb. I fay, BE : Bb or Ee : : Sine of the Incidence : Sine of the Refraction. For the Angle of Incidence ABP=BbE=FEe=EeQ, by Parallel Lines. And the Angle of Refraction QBE=BEF; but in the Oblique Triangle BbE or BEe, the Sides are as the Sines of their oppofite Angles; confequently the Side BE is to the Side Bb or Ee, as the Sine of the Angle BbE or BbF, is to the Sine of the Angle BEF =EBQ; that is, as the Sine of Incidence to the Sine of Refraction.

PROP. VI.



Let ABEF be any Veffel, on the Bottom of which (when empty) let there be laid any Object, as half a Crown, at G; and let it be required to find the Place (called the Focus) of its vifible Appearance by Refraction, when the Veffel is filled with Water to the Height DC; fuppofe GR a Ray of Light proceeding from the Object G to

G to the Surface of the Water at R, and is refracted into the Air; let fall the Perpendicular PQ and make as PL : SK : : 3 : 4, draw KH, then shall H be the visible Place of the Object, which may now be seen by an Eye at K, but could not before by much. Wherefore by the Refraction of Water the Bottoms of all Vessels appear projected at a Distance beyond their Real Place, as that Point of the Bottom G is projected to H; and it will thereby appear elevated also, or to rise up nearer the Eye, as the Point G will appear in the Perpendicular IG to be elevated to the Point M, to an Eye in I; and 'tis plain the Object G must be lifted up to the Point M, in order to be seen by the Eye at K, was there no Water in the Vessel.

Hence the Focus, or visible Place of the Object G to an Eye at K, will be in the Point H; and to an Eye in I, in the Point M; and the Bottom FE will appear elevated to the Height NO, and the visible Depth of the Water will be DN or CO. These Particulars may appear as so many Paradoxes, but are most easily proved by Experiment.

PROP. VII.

To shew the Trigonometrical Calculation *of the Difference between the True Place* G, *and the apparent or visible Place* H *or* M (viz. *the Quantity of* GH *and* GM,) *of any Object by Refraction.*

LET the Quantity of the Angle of Incidence out of Water into the Air GRQ, be 38° 00′; then shall the Angle HRQ=KRP be 55° 00′, or thereabouts; also let the Depth of the RQ be 2 Foot, or 24 Inches.

Then say by the Gunter;

As Radius : to the Tangent of 38°, on the Tangent-Line;

: : So is RQ = 24 Inches : to about $18\frac{3}{4}$ = GQ, on the Line of Numbers.

Then say again,

As the Radius : to the Tangent of 55°

: : So is RQ = 24 : to $34\frac{1}{2}$ Inches = QH.

Then QH − QG = GH = $15\frac{3}{4}$ Inches.

Again, in the Right-angled Triangle HGM, there is known the Side GH (last found) = $15\frac{3}{4}$ Inches; and the Angles, to find GM.

Say by the Gunter,

As Radius : to the Tangent of GHM = 35°

: : So is GH = $15\frac{3}{4}$: to GM = 11 Inches, nearly.

Whence the Eye at K will be deceived about $15\frac{3}{4}$ Inches, and that at I about 11 Inches; so that the greater the Obliquity of Sight, the greater is the Deception.

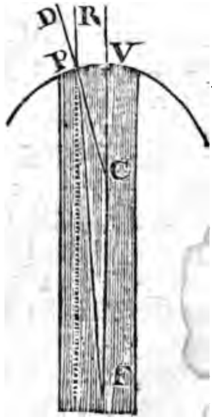
PROP. VIII.

Rays coming out of Air, and falling on a Convex Glass Superficies, in a Parallel Position to, and very near the Axis, converge in a Focal Point F, *which will be distant from the Vertex* V *very near three Semidiameters of the Sphere.*

NOW because the Refraction in Glass out of Air, is as 17 : 11, or as 3 : 2 very nearly; therefore take FV : FC : : Sine of Incidence : Sine of Refraction, that is, as 3 : 2 then shall F be the Focus, to which a Ray R parallel to, and near the Perpendicular, will converge. For PF : FC : : Sine of FCP = Sine of PCV = RDP the Angle of Incidence

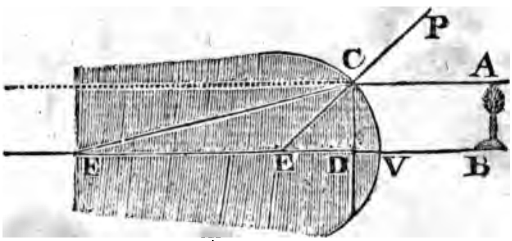
: the

: the Sine of FPC the Angle of Refraction; But FP : FC :: FV : FC; wherefore the Incident Ray RP shall be refracted into PF. But because FV : FC :: 3 : 2; therefore let FV be divided into three equal Parts, then shall FC be two of those Parts, and CV will be one; but CV is the Radius, therefore FC will be a Diameter; consequently FV will be distant from V, 3CV or 3 Semidiameters.



PROP. IX.

To calculate accurately the Focal Distance of Parallel Rays, refracted out of Air by the Spherical Convex Superficies of a Glass Lens.



LET AB be an Object, from which a Ray of Light AC proceeds parallel to the Axis of the Sphere FB, and impinges on the Convex Superficies of Glass in the Point C, from whence it is refracted to the Point F. Suppose the Angle of Incidence ACP $=42^\circ$, then it will be $17 : 11 ::$ Sine of 42°: Sine of $25^\circ\ 30'=$ECF, the Angle of Refraction, let the Height of the Object AB=CD, be 12 Inches, then say,

On the Sector.

As=Sine of ACP or VEC, 42° : || DC or AB 12; : : So=Radius : || EC, 18 ¼ Inches for the Semidiameter.

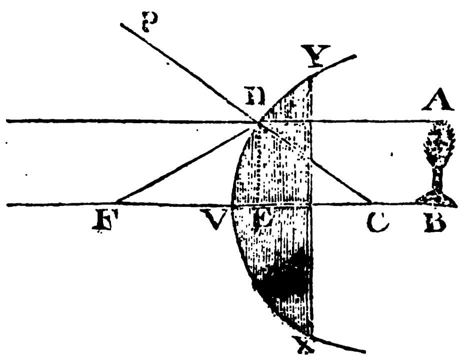
Say again,

As=Sine of CFE 16° 30′ : =Sine of FCE 25° 30′; : : So || EC 18 ¼ : || EF 27¼ Inches.

But FE+CE (=EV) =FV=45 ¼ Inches, the Distance of the Focus F from the Vertex of the Superficies V, as was to be found. Hence it appears that the Parallel Rays must be very near the Axis indeed, that their Focus may be the Distance of three Semidiameters from the Vertex V, according to the last Proposition.

PROP. X.

To calculate the Focal Distance of Parallel Rays falling on the Plane Surface of a Plano-convex Glass Lens.



LET AD be a Ray falling perpendicularly on the Plane Surface of the Lens YX, where it will suffer no Refraction in passing the Glass Medium, till it arrives to the Concave Surface at D, where it is refracted into the Air in the Direction DF, to the Focus

Focus in F. Now FD : FC : : Sine of FCD=ADC : Sine of PDF ; that is, as the Sine of the Incidence to the Sine of Refraction, out of Glafs into Air. Then fuppofing the Height of the Object AB 12 Inches, and the Incidence ADC=35° 30′, then the Angle of Refraction PDF will be about 66° 30′, then to find VC, and FC;

Say by the Sector;

As=Sine of DCV 35° 30′ : || AB or DE 12; : : So=Radius : || CD or CV 20½, for the Semidiameter.

Again for FV, *fay*;

As=Sine of DFC 31° : || DC 20½ Inches;
: : So=Sine of PDF 66° 30′ : || FC 36½ Inches.

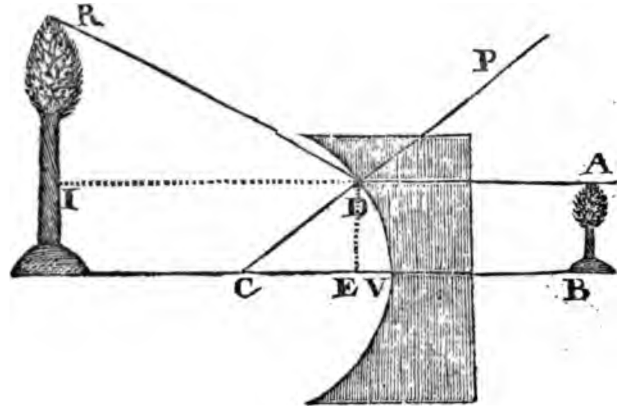
But FC—VC=FV=16 Inches, the Focal Diftance required. Hence the Focus of thofe Rays falling very near the Axis, will be diftant from V about the Diameter of the Sphere. The Converfe of this Propofition is eafy to be underftood.

PROP. XI.

THE Effect of Plano-concave Glaffes, is contrary to that of Plano-convex ones; as they converge the Rays, thefe dilate them; and in the fame Proportion that the Images of Objects are diminifhed in them, they are increafed or inlarged by thefe. The Proportion which, or Diftances at which (from the Point of Incidence D) the Bulk of any Object is increafed, is thus computed.

Suppofe the Height of the Object AB=12 Inches, and the Angle of Incidence ADP=PCB, be 35° 30′, as before; then the Angle of Refraction into the Air will be 66° 30′=CDR, and the Difference of thefe Angles, will be the Angle RDI, (by which the Images of Objects are increafed in thefe Sort of Lenfes) this Angle is 31 Degrees; and therefore fuppofe it were required to know how large the Object AB will

appear



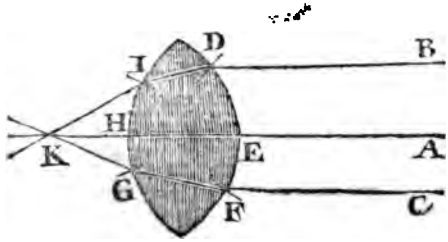
appear at the Diſtance DI, from the Point of Incidence, *viz.* 3 Foot, or 36 Inches, ſay,

As=Sine of IRD 59° : || ID 36 Inches.

: : So=Sine of IDR 31° : || IR 22 Inches; and therefore ſuch is the inlargement of the Object; to which add IS=AB its real Altitude, the Sum is SR=34 Inches, the apparent Length of the dilated Object, at the given Diſtance, as required.

PROP. XII.

IN a Convex Lens, the parallel Rays B, C, are refracted at twice (firſt, thro' the Glaſs from D, F, to I, G; and then again into the Air from I, G,) to the Point or Focus K; the ſame is affirmed of Rays which proceed from one Point only of the Object.



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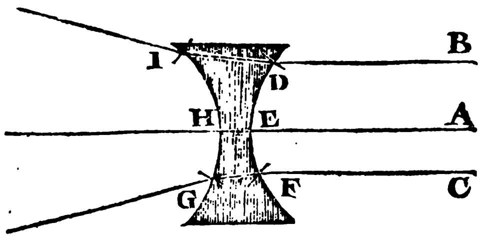
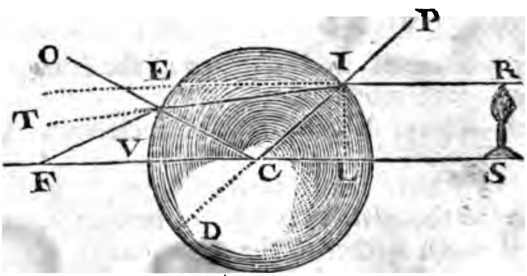
The direct contrary is effected by a Concave Lense; for therein after two Refractions at D, F, and I, G, the Rays aforesaid diverge and are dispersed, as the other were conveyed and collected in one Point.



PROP. XIII.

To calculate the Distance of the Focus F *of Parallel Rays refracted thro' a Sphere or Globe of Glass, from the Vertical Point* V, *or Center* C *of the Sphere.*



SUPPOSE IDE a Sphere of Glass; R a Ray of Light parallel to its Axis L V, and incident in the Point I, in an Angle PIR of 42°; RS any Object 12 Inches high. Then as 17 : 11 : : Sine of Incidence PIR 42° : Sine of Refraction CIE 25° 30'. But because of the Isocelels Triangle ICE, the Angle CIE=CEI; again, the Angle DCE=CIE+CEI=51°; but LCV=ICL,=PIR=42°; therefore the Angle

gle DCE—DCV=VCE=9°. Now because the Angle CEI=TEO, is the Angle of Incidence of the refracted Ray IE; therefore as 11 : 17 : : Sine of TEO : Sine of EF=42°; that is the Refraction out of Glass into Air, will always be equal to the first Angle of Incidence RIP. Therefore 180°—OEF (42°)= 138°. And thus in the Oblique Triangle FEC, all the Angles are known, and the Side CE (being the Semidiameter of the Sphere) will in this Case be $18\frac{1}{4}$ Inches, by *Prop.* IX. Wherefore to find the Side CF,

Say thus by the Sector,

As=Sine of CFE, 33° : || CE, $18\frac{1}{4}$ Inches,
: : So=Sine of OEF, 42° : || CF, $22\frac{1}{2}$ Inches.
Now CF—CV=VF=$4\frac{1}{4}$ Inches, as was required.
And in all Incidences whatsoever, it will be,

The Semidiameter of the Sphere CE,
Is to the Distance of the Focus F from the Center C, *viz.* CF;
As the Sine of the Difference of the double Angles of Incidence and Refraction,
To the Sine of the Angle of Incidence.

Thus I have finished what I design of the *Trigonometrical Calculations in Optics*; I might have advanced it much farther in the consideration of *Oblique Rays*, as well as parallel ones; but I have already overshot my Mark; and therefore shall stop here; having done more this way than I have seen done by any other; and shall leave the Remainder to the Learner's own Improvement.

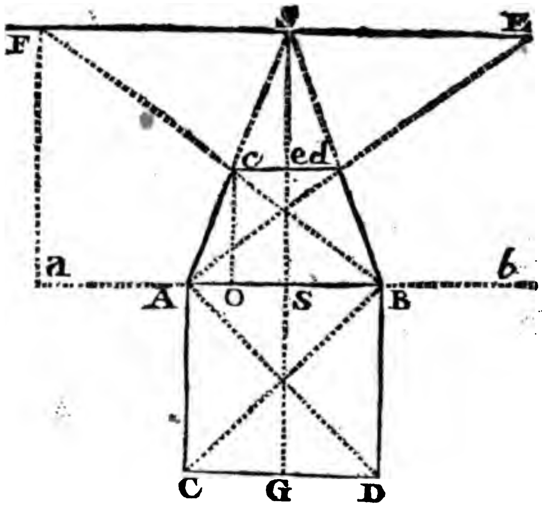
CHAP.

CHAP. X.

Plain Trigonometry *applied to* Perspective; *shewing how to delineate Objects, as they appear to the Sight; and to find the Proportion of the visible and real Magnitudes of distant Objects by Calculation.*

THE Art of Perspective is exceeding curious and delightful to all Persons who have any Notion or Taste of the Liberal Arts and Sciences, and is not only useful but indispensibly necessary for several Artificers, as Limners, Designers, Architects, &c. and it is therefore very much to be wonder'd at, that is no more common and cultivated than it is.

What I design here is not to give the Reader a Treatise of the Art in general, but only to give the young Student an Example or two of the Perspective Delineation of remote Objects; and also how the same Thing may be done by Trigonometrical Calculation; and how the Proportion of the visible and real Magnitude of Objects is to be determined by Sines and Tangents. [*Prop.* I. by Perspective.] In the first Place let it be required to delineate the Appearance of the Square ABCD, in a direct View to an Eye at the Height of 6 Feet above it, and at the Distance of 30 Feet, according to the Rules of Perspective.



I. Let ABCD be the given Square, each of whose Sides is 4 Feet or 48 Inches; and it is required to find its Perſpective Delineation for the aforeſaid given Height and Diſtance of the Eye.

II. In order to do this, by the Rules of Perſpective, draw the Baſe or Ground Line a b; this repreſents the Surface of the Earth or Ground on which we ſtand to view the Object ABCD. This Line is always fixed.

III. Draw the Viſual or Horizontal Line FE, in which the Point S, called the Point of Sight, is always found, becauſe it repreſents that Line which paſſeth thro' the Eye of the Beholder parallel to the Horizon; and therefore muſt be drawn equal to the Height of the Eye above the Ground, *viz.* 6 Feet, =Sa.

IV. From S ſet off both ways the Diſtance of the Eye from the Object, *viz.* 30 Feet=SF or SE; and for this Reaſon the two Points F, E, are called, the *Points of Diſtance.*

V. Draw

V. Draw up the Radial Lines AS and BS.

VI. Then draw the Diagonals FB and AE;

VII. *Lastly*, Join the Points of Intersection of the Radials and Diagonals c and d; then shall AcdB be the Perspective Appearance of the Square, as required.

PROP. II.

To find the same Perspective Appearance of the Square by Trigonometry.

I. IN order to this we must find the Dimunition and Position of the Sides or Lines Ac, c d, dB, and Se, or cO, and the Business will be done; thus,

II. In the Right-angled Triangle ASs, there's given the two Sides S s=6, and A s=2; to find the Angles.

By the Sector, say;

As || Ss, 6, : =Tangent Radius.
: : So || A s, 2, :=Tangent of ASs 18° 15′.

III. In the Right-angled Triangle FaB, there is known the Side Fa (=Ss) =6, and the Side aB (=FS+sB) =32, to find the Angles; *say*,

As || Ba, 32, : =Tangent Radius,
: : So || Fa, 6, : =Tangent of FBa, 10° 40′.

IV. In the Oblique Triangle AcB, there are known all the Angles, by the two last Operations, and the Side AB=4; to find the other two Sides Ac, and cB, *say*,

As=Sine of ScB, 82° 25′, : || AB, 4 Feet;
: : So=Sine of cBA, 10° 40′ : || Ac, O, 75 of a Foot;
: :So=Sine of BAc, 71° 45′ : || Bc, 3.83 Foot.

V. In the Right-angled Triangle AOc, there are known the Side Ac=0.75, and the Angles, to find the other two Sides AO, and Oc, thus;

As=Radius : || Ac, 0.75 of a Foot, or 9 Inches, : : So=Sine of OAc 71° 45′, : || Oc, 8½ Inches.
: : So=Sine of AcO 18° 15′ : || AO 2.85. Inches.

But sA – AO=SO=ce=21.15 Inches and 2ce=cd =42.3 Inches.

VI. Thus have you obtain'd by Calculation, the Position and Quantity of each Side or Line in the Perspective or apparent Square; which compared with the real Magnitudes, the Difference or Diminution will the better appear thus, in Inches, and Decimal Parts.

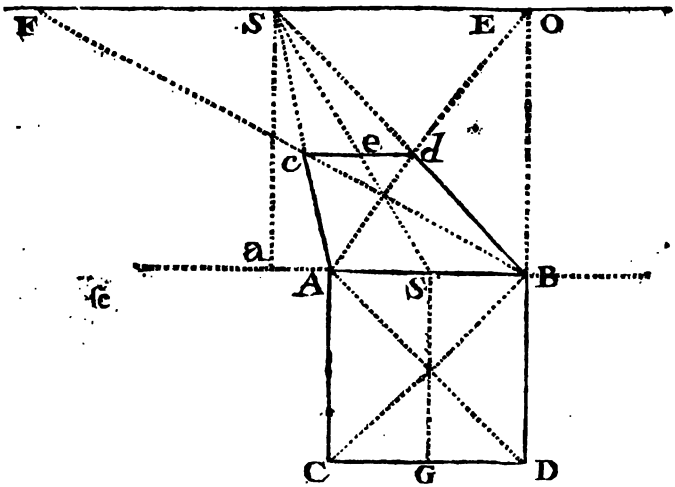
	Real Mag.	Persp. Mag.	Diminution.
The Front	AB=48	AB=48	0.00
The Side	AC=48	A c= 9	37.00
The Diagonal	AD=68	A d=46	22.00
The Side	CD=48	c d=42.3	5.7
Middle Line	sG=48	S e=8.5	39.5

And such are the Dimensions and the Differences of a Geometrical and Perspective Square on the Conditions above given, in a direct View: In the next Place let us see the Difference and Alteration made by an *Oblique View*.

PROP. III.

To represent the afore-mentioned Square perspectively in an Oblique View.

LET ABCD be the Square, as before, to be put into Perspective obliquely; and let the Angle AsS be the Obliquity of the View, which suppose to be



be 60 Degrees. Draw the Line Ss, ſo ſhall S be the Point of Sight. Draw up the Radials AS and BS; and the Diagonals FB and AE; then join the Points of Interſection c, d, by the Traverſe Line cd; thus ſhall AcdB the Perſpective View of the Square in the given Oblique Poſition of the Eye, as was required.

PROP. IV.

To find the Dimenſions and Poſition of the Square, in the given Oblique View *in* Perſpective, *by* Trigonometry.

I. LET the Dimenſions of the Geometrical Square, the Height of the Eye, the Points of Diſtance, and the Angle of *Oblique View*, be all the ſame as has been above-ſtated.

II. Then

II. Then in Right Triangle a s S, there are known all the Angles and the Side aS=6; to find the Side a s, say;

As=Tangent Radius : || aS, 6 Feet.
So=Tangent of aSs, 30°, || a s, 3,5 Feet; hence aA=1.5.
So=Secant of aSs, 30°, || Ss, 7.1 Feet.

III. In the Right Triangle Aas, there is given the Side aA=1.5, and the Side aS=6, to fin om An-gles, and the Side SA, say thus;

As || aS, 6 : =Tangent Radius.
So || aA 15 : =Tangent of aSA, 13° 50′.

Then say,

As=Sine of aSA 13° 50′ : || aA 15 Feet.
So=Radius : || SA, 6.2 Feet.

IV. Again, in the Right Triangle BOF, there is given the Side BO=6, and the Side FO=30+5.5=35.5; to find the Angle BFO, say thus;

As || FO, 35.5 : Tangent Radius.
So || BO 6 : =Tangent of BFO, 9° 35′,=ABc.

V. In the next Place, in the Oblique Triangle AcB, there are known all the Angles, and the Side AB=4; to find the other two Sides Ac, and cB, say thus;

As=Sine AcB, 66° 35′ : || AB, 4 Feet, or 48 Inches.
So=Sine cBA, 9° 35′, : || Ac, 0.74 Feet, or [illegible] Inches.
So=Sine aAS, 76° 10′ : || Bc, 4.25 Feet, or [illegible] Inches.

VI. Then because of Similar Triangles ASB and cSd, it will be as AS=6.2 : AB=4 : : Sc=5.46 : cd=3.36 Feet, and AS=6.2 : Ss=7.1 : : Sc=5 46

: Se

∴ Se=6.25. But Ss—Se=se=0.85 Foot, or 10.1 Inches. Also, as ed=1.68 : sB=2 : : Se=0.85 : Bd=1.02 Foot.

VII. *Lastly*, In the Oblique Triangle Acd, there is known the Side Ac=0.74; the Side cd=3.36; and the included Angle Acd=cAa=76°.10′; thence the Side Ad will be found 3.25. And thus are all the Sides and their Positions found in the Perspective Square.

These [illegible] ith respect to their Originals in the Geomet[illegible] e, stand thus (with the Diminution of each respectively) in Inches.

	Real Mag.	Persp. Mag.	Dimunitions
The Front Line	AB=48	AB=48	00.00
The Side	AC=48	A c=8.9	39.10
The Side	BD=48	B d=12.24	35.76
The Side	CD=48	c d=40.22	7.78
The Line	G s=48	s e=10	38.00
The Diagonal	AD=68	Ad=39	29.00
The Diagonal	BC=68	B c=51	17.00

Thus the young Trigonometer may at his Pleasure find by the Rules of his Art, the true Dimensions, Proportions, and Diminutions of the several Lines or Parts of the Perspective Appearance of a Square in a direct and *Oblique View*; and that to a Mathematical Exactness and Nicety.

Also, since by means of a Square, various other plain Figures are put into Perspective, as a Triangle, Parallelogram, Rhombus, any Regular Polygon, *&c.* 'tis easy to apprehend how Trigonometry may be extended to the Calculation of their Dimensions in Perspective likewise.

I have

I have given this Example or two in Perſpective, only becauſe the young Artiſt may be the better apprized of the extenſive Uſe of this excellent Art, and that it may be applied to anſwer ſeveral both curious and uſeful Purpoſes in this delightful Art of Perſpective; and I have the rather done this, becauſe I have not ſeen it thus applied by any other Hand.

to fin

F I N I S.